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From a different point of view to near-rings: Soft zero-symmetric and constant parts of soft near-rings with applications

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Abstract: In this study, we describe soft parts, named soft 0-symmetric and soft constant parts, of soft intersection near-rings and soft union near-rings, and obtain their fundamental features. We explore the relations between the parts of near-rings and soft parts of soft intersection near-rings and soft union near-rings, and we give some applications of these parts to soft sets. Additionally, soft intersection (union) product of soft intersection (union) near-ring are introduced and applied on soft parts of soft intersection near-rings and soft union near-rings, respectively.

Key words: Soft set, soft intersection near-ring, soft union near-ring, soft zero symmetric part, soft constant par

1. Introduction

Soft set theory was introduced by Molodtsov [1] as a way to model vagueness and uncertainty. Since then, work on the theory has advanced quickly and found a wide range of applications, including the mean of algebraic structures as in [2–23], as well as the structures and operations of soft sets as in [28–46]. Additionally, the theory of soft sets keeps growing and diversifying tremendously in the mean of soft decision making, as demonstrated by the papers listed below [24–27, 47–49, 51].

Sezgin et al. [52] defined a new concept, soft intersection near-ring, by utilizing soft sets and the intersection operation of sets. This concept combines near-ring theory, set theory, and soft set theory, making it highly useful for obtaining results in the mean of near-ring structure. They provide several applications of soft int near-rings to near-ring theory based on the definition. Additionally, Sezgin et al. [53] established a new concept, soft union near-ring, and provided some applications of soft uni near-rings to near-ring theory by utilizing soft sets and the intersection operation of sets.

Manikantan et al. [50] introduced the concepts of soft zero-symmetric near-ring, soft constant near-ring, soft near-field and soft Q- simple near-ring over a near-ring and investigated the properties of these notions with illustrative example in a different manner. In this paper, we try to convey the concept of 0-symmetric and constant parts of a near-ring to soft intersection near-rings and soft union near-rings as soft 0-symmetric and soft constant parts, and derive their basic properties. We give some applications of these soft parts to soft sets and study relations between the parts of near-rings and soft parts of soft int near-rings and soft union near-rings. Finally, we define soft int (uni) product of soft intersection (union) near-ring and give the applications of them on soft parts of soft intersection near-rings and soft union near-rings, respectively. In a near-ring structure,

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0-symmetric and constant parts are of great importance for chracterization of the near-ring; so constructing the soft 0-symmetric part and soft constant part help the soft near-ring theory to improve.

2. Preliminaries

In this section, we recall some basic notions relevant to near-rings and soft sets. By a *near-ring*, we shall mean an algebraic system $(N,+,.)$, where

N1) $(N,+)$ forms a group (not necessarily abelian)

N2) $(N,.)$ forms a semi-group and

N3) $(a+b)c = ac+bc$ for all $a,b,c \in N$ (i.e. we study on right near-rings.)

Throughout this paper, N will always denote a right near-ring with zero element 0. The subset $\{n \in N : n0 = 0\}$ of N, is called a zero-symmetric part of N and denoted by N_0 and the subset $\{n \in N : n0 = n\}$ of N, is called a constant part of N and denoted by N_c. For all undefined concepts and notions regarding near-rings and semi-nearring, we refer to [54, 55]. From now on, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A,B,C \subseteq E$.

Definition 2.1. [1, 48] A soft set ξ_A over U is a set defined by

$$\xi_A : E \to P(U) \text{ such that } \xi_A(x) = \emptyset \text{ if } x \notin A.$$

Here, ξ_A is also called *approximate function.* A soft set over U can be represented by the set of ordered pairs

$$\xi_A = \{(x, \xi_A(x)) : x \in E, \xi_A(x) \in P(U)\}.$$

It is clear to see that a soft set is a parametrized family of subsets of the set U. It is worth noting that the sets $\xi_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. We refer to [1, 28, 48] for further details.

Definition 2.2. [48] Let ξ_A and ξ_B be soft sets over U. Then, *union* of ξ_A and ξ_B, denoted by $\xi_A \widetilde{\cup} \xi_B$, is defined as $\xi_A \widetilde{\cup} \xi_B = \xi_{A\widetilde{\cup}B}$, where $\xi_{A\widetilde{\cup}B}(x) = \xi_A(x) \cup \xi_B(x)$ for all $x \in E$.

Intersection of ξ_A and ξ_B, denoted by $\xi_A \widetilde{\cap} \xi_B$, is defined as $\xi_A \widetilde{\cap} \xi_B = f_{A\widetilde{\cap}B}$, where $f_{A\widetilde{\cap}B}(x) = \xi_A(x) \cap \xi_B(x)$ for all $x \in E$.

Definition 2.3. [48] Let ξ_A and ξ_B be soft sets over U. Then, $\vee$*-product* of ξ_A and ξ_B, denoted by $\xi_A \vee \xi_B$, is defined as $\xi_A \vee \xi_B = \xi_{A\vee B}$, where $\xi_{A\vee B}(x,y) = \xi_A(x) \cup \xi_B(y)$ for all $(x,y) \in E \times E$.

$\wedge$*-product* of ξ_A and ξ_B, denoted by $\xi_A \wedge \xi_B$, is defined as $\xi_A \wedge \xi_B = f_{A\wedge B}$, where $\xi_{A\wedge B}(x,y) = \xi_A(x) \cap \xi_B(y)$ for all $(x,y) \in E \times E$.

Definition 2.4. [29] Let ξ_A and ξ_B be soft sets over U. Then, *restricted union* of ξ_A and ξ_B, denoted by $\xi_A \cup_{\mathcal{R}} \xi_B$, is defined defined as $\xi_A \cup_{\mathcal{R}} \xi_B = \xi_{A\cup_{\mathcal{R}}B}$, where $\xi_{A\cup_{\mathcal{R}}B}(x) = \xi_A(x) \cup \xi_B(x)$ for all $x \in A \cap B \neq \emptyset$.

Restricted intersection of ξ_A and ξ_B, denoted by $\xi_A \Cap \xi_B$, is defined as $\xi_A \Cap \xi_B = f_{A\Cap B}$, where $\xi_{A\Cap B}(x) = \xi_A(x) \cap \xi_B(x)$ for all $x \in A \cap B \neq \emptyset$.

Definition 2.5. ([52]) Let N be a near-ring and ξ_N be a soft set over U. Then, ξ_N is called *soft int near-ring* over U if

i) $\xi_N(x+y) \supseteq \xi_N(x) \cap \xi_N(y)$

ii) $\xi_N(-x) = \xi_N(x)$

iii) $\xi_N(xy) \supseteq \xi_N(x) \cap \xi_N(y)$

for all $x, y \in N$.

From now on, soft int near-ring is designated by SIN.

Proposition 2.1. *[52] Let ξ_N be a SIN over U. Then, $\xi_N(0) \supseteq \xi_N(x)$ for all $x \in N$.*

Theorem 2.1. *[52] If ξ_N and ξ_M are SINs over U, then so is $\xi_N \wedge \xi_M$ over U.*

Definition 2.6. [52] Let ξ_N, g_M be SINs over U. Then, the *product of SINs* ξ_N and g_M is defined as $\xi_N \times g_M = h_{N\times M}$, where $h_{N\times M}(x,y) = \xi_N(x) \times g_M(y)$ for all $(x,y) \in N \times M$.

Theorem 2.2. *[52] If ξ_N and g_M are SINs over U, then so is $\xi_N \times g_M$ over $U \times U$.*

Theorem 2.3. *[52] If ξ_N and h_N are two SINs over U, then so is $\xi_N \widetilde{\cap} h_N$ over U.*

Definition 2.7. [53] Let N be a near-ring and ξ_N be a soft set over U. Then, ξ_N is called a *soft uni near-ring* over U if it satisfies the following properties:

i) $\xi_N(x+y) \subseteq \xi_N(x) \cup \xi_N(y)$,

ii) $\xi_N(-x) = \xi_N(x)$,

iii) $\xi_N(xy) \subseteq \xi_N(x) \cup \xi_N(y)$

for all $x, y \in N$.

From now on, soft uni near-ring is designated by SUN and near-ring by NR.

Proposition 2.2. *[53] Let ξ_N be a SUN over U. Then, $\xi_N(0) \subseteq \xi_N(x)$ for all $x \in N$.*

Theorem 2.4. *[53] If ξ_N and ξ_M are SUNs over U, then so is $\xi_N \vee \xi_M$ over U.*

Theorem 2.5. *[53] If ξ_N and h_N are two SUNs over U, then so is $\xi_N \widetilde{\cup} h_N$ over U.*

3. Soft parts of soft int near-rings

Now, we define soft zero-symmetric and soft constant parts of SINs.

Definition 3.1. Let ξ_N be a SIN over U and $M \subseteq N$. Let ξ_M be the restricted function of f to M, i.e. for all $x \in M$ $\xi_M(x) = \xi_N(x)$. If M is a maximal subset of N such that

i) For all $m \in M$, $\xi_M(m0) = \xi_M(0)$, then the soft set ξ_M is called the soft zero-symmetric part of ξ_N and denoted by $(F_N)_0$.

Table 1. Addition and Multiplication Tables of N

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	0
c	a	a	c	a

ii) For all $m \in M$, $\xi_M(m0) = \xi_M(m)$, then the soft set ξ_M is called the soft constant part of ξ_N and denoted by $(\xi_N)_c$.

Example 3.1. *Let the NR $(N, +, .)$ be defined on the Klein's four group $N = \{0, a, b, c\}$ as following [54].*

Assume that N is the set of parameters and $U = \left\{ \begin{bmatrix} x & x \\ y & y \end{bmatrix} \mid x, y \in \mathbb{Z}_4 \right\}$, 2×2 matrices with $\mathbb{Z}_4$ terms, is the universal set. We construct a soft set ξ_N over U by

$$\xi_N(0) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \right\}$$

$$\xi_N(a) = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \right\}$$

$$\xi_N(b) = \xi_N(c) = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \right\}.$$

Then, one can easily show that ξ_N is a SIN over U. Since $M = \{0, b\}$ is a maximal subset of N such that $\xi_M(m0) = \xi_M(0)$ for all $m \in M$, then $(\xi_N)_0 = \xi_M$. Similarly, since $K = \{0, a\}$ is a maximal subset of N such that $\xi_K(k0) = \xi_K(k)$ for all $k \in K$, then $(\xi_N)_c = \xi_K$.

If $N = N_0$ or $N = N_c$, the soft zero-symmetric part of ξ_N or soft constant part of ξ_N are easily obtained by following theorem:

Theorem 3.1. *Let ξ_N be a SIN over U.*

i) If $N = N_0$, then $(\xi_N)_0 = \xi_N$.

ii) If $N = N_c$, then $(\xi_N)_c = \xi_N$.

Proof. **i)** If $N = N_0$, then $n0 = 0$ for all $n \in N$. Hence $\xi_N(n0) = \xi_N(0)$ for all $n \in N$. Therefore $(\xi_N)_0 = \xi_N$.
ii) If $N = N_c$, then $n0 = n$ for all $n \in N$. Hence $\xi_N(n0) = \xi_N(n)$ for all $n \in N$. Therefore $(\xi_N)_c = \xi_N$. □

The converse of Theorem 3.1 doesn't hold, in general. We have the following example:

Example 3.2. *Let the NR $(N, +, .)$ be defined on the Klein's four group $N = \{0, a, b, c\}$ as in Table 2. Assume that N is the set of parameters and $U = Z_5$ is the universal set. We construct a soft set ξ_N over U by $\xi_N(0) = \xi_N(a) = Z_5$, $\xi_N(b) = \xi_N(c) = \{0, 1, 2\}$. The one can easily show that ξ_N is a SIN over U. Since $\xi_N(a0) = \xi_N(a) = \xi_N(0)$, $\xi_N(b0) = \xi_N(0)$ and $\xi_N(c0) = \xi_N(a) = \xi_N(0)$, then $(\xi_N)_0 = \xi_N$, but $a0 \neq 0$, i.e. N*

Table 2. Addition and Multiplication Tables of N

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a	a	c	c

is not a zero-symmetric NR*.*
If we define another soft set ζ_N *over* U *by* $\zeta_N(0) = \zeta_N(a) = \zeta_N(b) = \zeta_N(c) = \{0,1,2,3\}$*, then* ζ_N *is a* SIN *over* U*. Since* $\zeta_N(n0) = \zeta_N(n)$ *for all* $n \in N$*,* $(\zeta_N)_c = \zeta_N$*. But* $b0 = 0 \neq b$*, i.e.* N *is not a constant* NR*.*

Theorem 3.2. *Let* N *and* M *be* NR*s and let* ξ_N *and* ζ_M *be* SIN*s over* U*. Then,*

i) $(\xi_N)_0 \wedge (\zeta_M)_0 = (\xi_N \wedge \zeta_M)_0$

ii) $(\xi_N)_c \wedge (\zeta_M)_c = (\xi_N \wedge \zeta_M)_c$

Proof. **i)** By Theorem 2.1, $\xi_N \wedge \zeta_M$ is a SIN over U. Assume that $(\xi_N)_0 = \xi_K$ and $(\zeta_M)_0 = \zeta_L$. Then K is a maximal subset of N such that $\xi_K(x) = \xi_N(x)$ for all $x \in K$ and L is a maximal subset of M such that $\zeta_L(x) = \zeta_M(x)$ for all $x \in L$. By Definition 2.3, let $\xi_K \wedge \zeta_L = t_{K\wedge L}$, where $t_{K\wedge L}(x,y) = \xi_K(x) \cap \zeta_L(y)$ for all $(x,y) \in K \times L$. Let $(k,l) \in K \times L$. Then,

$$\begin{aligned}(\xi_K \wedge \zeta_L)((k,l).(0,0)) &= t_{K\wedge L}(k0, l0)\\ &= \xi_K(k0) \cap \zeta_L(l0)\\ &= \xi_K(0) \cap \zeta_L(0)\\ &= t_{K\wedge L}(0,0)\end{aligned}$$

Hence, $\xi_K \wedge \zeta_L = (\xi_N)_0 \wedge (\zeta_M)_0 = (\xi_N \wedge \zeta_M)_0$. The rest of the proof can be obtained similarly. □

Theorem 3.3. *Let* N *and* M *be* NR*s and let* ξ_N *and* ζ_M *be* SIN*s over* U*. Then,*

i) $(\xi_N)_0 \times (\zeta_M)_0 = (\xi_N \times \zeta_M)_0$

ii) $(\xi_N)_c \times (\zeta_M)_c = (\xi_N \times \zeta_M)_c$

Proof. **i)** By Theorem 2.2, $\xi_N \times \zeta_M$ is a SIN over $U \times U$. Assume that $(\xi_N)_0 = \xi_K$ and $(\zeta_M)_0 = \zeta_L$. Then, K is a maximal subset of N such that $\xi_K(x) = \xi_N(x)$ for all $x \in K$ and L is a maximal subset of M such that $\zeta_L(x) = \zeta_M(x)$ for all $x \in L$. By Definition 2.6, let $\xi_K \times \zeta_L = t_{K\times L}$, where $t_{K\times L}(x,y) = \xi_K(x) \times \zeta_L(y)$ for all $(x,y) \in K \times L$. Let $(k,l) \in K \times L$. Then

$$\begin{aligned}(\xi_K \times \zeta_L)((k,l).(0,0)) &= t_{K\times L}(k0, l0)\\ &= \xi_K(k0) \times \zeta_L(l0)\\ &= \xi_K(0) \times \zeta_L(0)\\ &= t_{K\times L}(0,0)\end{aligned}$$

Hence, $\xi_K \times \zeta_L = (\xi_N)_0 \times (\zeta_M)_0 = (\xi_N \times \zeta_M)_0$. The rest of the proof can be obtained similarly. □

Theorem 3.4. *Let ξ_N and ζ_N be SINs over U. Then,*

i) $(\xi_N)_0 \widetilde{\cap} (\zeta_N)_0 = (\xi_N \widetilde{\cap} \zeta_N)_0$

ii) $(\xi_N)_c \widetilde{\cap} (\zeta_N)_c = (\xi_N \widetilde{\cap} \zeta_N)_c$

Proof. **i)** By Theorem 2.3, $\xi_N \widetilde{\cap} \zeta_N$ is a SIN over U. Assume that $(\xi_N)_0 = \xi_K$ and $(\zeta_N)_0 = \zeta_L$. Then, K is a maximal subset of N such that $\xi_K(x) = \xi_N(x)$ for all $x \in K$ and L is a maximal subset of N such that $\zeta_L(x) = \zeta_N(x)$ for all $x \in L$. By Definition 2.2, let $\xi_K \widetilde{\cap} \zeta_L = t_{K \widetilde{\cap} L}$, where $t_{K \widetilde{\cap} L}(x) = \xi_K(x) \cap \zeta_L(x)$ for all $x \in N$. Let $x \in N$. Then,

$$\begin{aligned}
(\xi_K \widetilde{\cap} \zeta_L)(x0) &= t_{K \widetilde{\cap} L}(x0) \\
&= \xi_K(x0) \cap \zeta_L(x0) \\
&= \xi_K(0) \cap \zeta_L(0) \\
&= t_{K \widetilde{\cap} L}(0)
\end{aligned}$$

Hence, $\xi_K \widetilde{\cap} \zeta_L = (\xi_N)_0 \widetilde{\cap} (\zeta_N)_0 = (\xi_N \widetilde{\cap} \zeta_N)_0$. The rest of the proof can be obtained similarly. □

Corollary 3.1. *Let ξ_N and ζ_N be SINs over U. Then,*

i) $(\xi_N)_0 \Cap (\zeta_N)_0 = (\xi_N \Cap \zeta_N)_0$

ii) $(\xi_N)_c \Cap (\zeta_N)_c = (\xi_N \Cap \zeta_N)_c$

Proof. The proof is similar to proof of Theorem 3.4, hence omitted. □

4. Soft parts of soft uni near-rings

Now, we define soft zero-symmetric and soft constant parts of SUNs.

Definition 4.1. Let ξ_N be a SUN over U and $M \subseteq N$. Let ξ_M be the restricted function of f to M, i.e. for all $x \in M$ $\xi_M(x) = \xi_N(x)$. If M is a maximal subset of N such that

i) For all $m \in M$, $\xi_M(m0) = \xi_M(0)$, then the soft set ξ_M is called the soft zero-symmetric part of ξ_N and denoted by $(\xi_N)_0$.

ii) For all $m \in M$, $\xi_M(m0) = \xi_M(m)$, then the soft set ξ_M is called the soft constant part of ξ_N and denoted by $(\xi_N)_c$.

Example 4.1. *Consider the additive group $(Z_6, +)$. Under a multiplication given in the Table 3, $N = (Z_6, +, .)$ is a (right) NR [56].*
Let $N = \mathbb{Z}_6$ be the set of parameters and $U = \mathbb{Z}^+$ be the universal set. We define a soft set ξ_N over U by

$$\begin{aligned}
&\xi_N(0) = \{2, 4\}, \\
&\xi_N(1) = \xi_N(5) = \{2, 4, 6, 8, 10\}, \\
&\xi_N(3) = \{2, 4, 8, 10\}, \\
&\xi_N(2) = \xi_N(4) = \{2, 4, 6, 10\}.
\end{aligned}$$

Then, ξ_N is a SUN over U. Since $M = \{0, 2, 4\}$ is a maximal subset of N such that $\xi_M(m0) = \xi_M(0)$ for all $m \in M$, then $(\xi_N)_0 = \xi_M$. Similarly, since $K = \{0, 3\}$ is a maximal subset of N such that $\xi_K(k0) = \xi_K(k)$ for all $k \in K$, then $(\xi_N)_c = \xi_K$.

Table 3. Multiplication Table of N

.	0	1	2	3	4	5
0	0	0	0	0	0	0
1	3	1	5	3	1	5
2	0	2	4	0	2	4
3	3	3	3	3	3	3
4	0	4	2	0	4	2
5	3	5	1	3	5	1

Theorem 4.1. *Let ξ_N be a SUN over U.*

i) If $N = N_0$, then $(\xi_N)_0 = \xi_N$.

ii) If $N = N_c$, then $(\xi_N)_c = \xi_N$.

Proof. Similar to proof of Theorem 3.1, hence omitted. □

The converse of Theorem 4.1 doesn't hold, in general. We have the following example:

Example 4.2. *Let the NR $N = (Z_6, +, .)$ be in Example 4.1. Assume that N is the set of parameters and $U = Z$, is the universal set. If we define a soft set F_N over U by $\xi_N(0) = \xi_N(1) = \xi_N(2) = \xi_N(3) = \xi_N(4) = \xi_N(5) = Z$, then ξ_N is a SUN over U and $(\xi_N)_0 = \xi_N$, but $3.0 \neq 0$, i.e. N is not a zero-symmetric NR. If we define another soft set ζ_N over U by $\zeta_N(0) = \zeta_N(2) = \zeta_N(4) = \{0,1,2\}$ and $\zeta_N(1) = \zeta_N(3) = \zeta_N(5) = \{0,1,2,3,4\}$, then ζ_N is a SUN over U. Since $\zeta_N(00) = \zeta_N(0)$, $\zeta_N(10) = \zeta_N(3) = \zeta_N(1)$, $\zeta_N(20) = \zeta_N(0) = \zeta_N(2)$, $\zeta_N(30) = \zeta_N(3)$, $\zeta_N(40) = \zeta_N(0) = \zeta_N(4)$ and $\zeta_N(50) = \zeta_N(3) = \zeta_N(5)$, then $(\zeta_N)_c = \zeta_N$. But $1.0 = 3 \neq 1$, i.e. N is not a constant NR.*

Theorem 4.2. *Let N and M be NRs and let ξ_N and g_M be SUNs over U. Then,*

i) $(\xi_N)_0 \vee (g_M)_0 = (\xi_N \vee g_M)_0$

ii) $(\xi_N)_c \vee (g_M)_c = (\xi_N \vee g_M)_c$

Proof. **i)** By Theorem 2.4, $\xi_N \vee g_M$ is a SUN over U. Assume that $(\xi_N)_0 = \xi_K$ and $(g_M)_0 = \zeta_L$. Then, K is a maximal subset of N such that $\xi_K(x) = \xi_N(x)$ for all $x \in K$ and L is a maximal subset of M such that $\zeta_L(x) = g_M(x)$ for all $x \in L$. By Definition 2.3, let $\xi_K \vee \zeta_L = t_{K\vee L}$, where $t_{K\vee L}(x,y) = \xi_K(x) \cup \zeta_L(y)$ for all $(x,y) \in K \times L$. Let $(k,l) \in K \times L$. Then

$$
\begin{aligned}
(\xi_K \vee \zeta_L)((k,l).(0,0)) &= t_{K\vee L}(k0, l0) \\
&= \xi_K(k0) \cup \zeta_L(l0) \\
&= \xi_K(0) \cup \zeta_L(0) \\
&= t_{K\vee L}(0,0)
\end{aligned}
$$

Hence, $\xi_K \vee \zeta_L = (\xi_N)_0 \vee (g_M)_0 = (\xi_N \vee g_M)_0$. The rest of the proof can be obtained similarly. □

Theorem 4.3. *Let ξ_N and ζ_N be SUNs over U. Then,* *i)* $(\xi_N)_0\widetilde{\cup}(\zeta_N)_0=(\xi_N\widetilde{\cup}\zeta_N)_0$

ii) $(\xi_N)_c\widetilde{\cup}(\zeta_N)_c=(\xi_N\widetilde{\cup}\zeta_N)_c$

Proof. **i)** By Theorem 2.5, $\xi_N\widetilde{\cup}\zeta_N$ is a SIN over U. Assume that $(\xi_N)_0=\xi_K$ and $(\zeta_N)_0=\zeta_L$. Then, K is a maximal subset of N such that $\xi_K(x)=\xi_N(x)$ for all $x\in K$ and L is a maximal subset of N such that $\zeta_L(x)=\zeta_N(x)$ for all $x\in L$. By Definition 2.2, let $\xi_K\widetilde{\cup}\zeta_L=t_{K\widetilde{\cup}L}$, where $t_{K\widetilde{\cup}L}(x)=\xi_K(x)\cup\zeta_L(x)$ for all $x\in N$. Let $x\in N$. Then,

$$\begin{aligned}(\xi_K\widetilde{\cup}\zeta_L)(x0) &= t_{K\widetilde{\cup}L}(x0)\\ &= \xi_K(x0)\cup\zeta_L(x0)\\ &= \xi_K(0)\cup\zeta_L(0)\\ &= f_{K\widetilde{\cup}L}(0)\end{aligned}$$

Hence, $\xi_K\widetilde{\cup}\zeta_L=(\xi_N)_0\widetilde{\cup}(\zeta_N)_0=(\xi_N\widetilde{\cup}\zeta_N)_0$. The rest of the proof can be obtained similarly. □

Corollary 4.1. *Let ξ_N and ζ_N be SUNs over U. Then,*

i) $(\xi_N)_0\cup_{\mathcal{R}}(\zeta_N)_0=(\xi_N\cup_{\mathcal{R}}\zeta_N)_0$

ii) $(\xi_N)_c\cup_{\mathcal{R}}(\zeta_N)_c=(\xi_N\cup_{\mathcal{R}}\zeta_N)_c$

Proof. The proof is similar to proof of Theorem 4.3. □

5. Soft int-product applied on soft parts of SINs

Definition 5.1. Let N be a NR and ξ_N and ζ_N be soft sets over the common universe U. Then, *soft int-product* $\xi_N\circ\zeta_N$ is defined by

$$(\xi_N\circ\zeta_N)(x)=\begin{cases}\bigcup_{x=yz}\{\xi_N(y)\cap\zeta_N(z)\}, & \text{if } \exists y,z\in N \textit{ such that } x=yz,\\ \emptyset, & \textit{otherwise}\end{cases}$$

for all $x\in N$. It is obvious that if N is a NR with identity, then the second condition does not exist.

Let ξ_N and ζ_N be soft sets over U. If $\xi_N(x)\subset\zeta_N(x)$ for all $x\in N$, then we denote it by

$$\xi_N\subset\zeta_N.$$

It is well-known that $N_0N_0\subseteq N_0$. Similarly, we have the following theorem:

Theorem 5.1. *Let ξ_N be a SIN and $(\xi_N)_0=\xi_M$. Then,*

$$\xi_M\circ\xi_M\subset\xi_M.$$

Proof. Let $m\in M$. If $(\xi_M\circ\xi_M)(m)=\emptyset$, then it is obvious that

$$\xi_M\circ\xi_M(m)\subseteq\xi_M(m),$$

for all $m\in M$. Thus,

$$\xi_M\circ\xi_M\subset\xi_M.$$

Otherwise, there exist elements $x, y \in M$ such that $m = xy$. Thus,

$$\begin{aligned}(\xi_M \circ \xi_M)(m) &= \bigcup_{m=xy} \{\xi_M(x) \cap \xi_M(y)\} \\ &\subseteq \bigcup_{m=xy} \xi_M(xy) \\ &= \bigcup \xi_M(m) \\ &= \xi_M(m)\end{aligned}$$

Thus,

$$\xi_M \circ \xi_M \subset \xi_M.$$

Here, note that since ξ_N is a SIN, $\xi_N(xy) \supseteq \xi_N(x) \cap \xi_N(y)$ for all $x \in N$. And, since ξ_M is the restricted function of f to M, then $\xi_M(xy) \supseteq \xi_M(x) \cap \xi_M(y)$. That is, ξ_M is a SIN, too. □

Note that the converse of Theorem 5.1, that is $\xi_M \subset \xi_M \circ \xi_M$, does not hold as seen in the following example.

Example 5.1. *[52] Let $N = \{0, 1, 2, 3\}$ be the (right) NR due to [54] (Near-rings of low order (D-10)) defined by the following tables:*

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	1	2	3
0	0	0	0	0
1	0	1	2	1
2	0	2	0	2
3	0	3	2	3

Assume that N is the set of parameters and $U = \left\{ \begin{bmatrix} x & x \\ y & y \end{bmatrix} \mid x, y \in \mathbb{Z}_4 \right\}$, 2×2 matrices with $\mathbb{Z}_4$ terms, is the universal set. We define a soft set ξ_N over U by

$$\xi_N(0) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 3 \end{bmatrix} \right\}$$

$$\xi_N(1) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \right\},$$

$$\xi_N(2) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\},$$

$$\xi_N(3) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \right\}.$$

Then, one can easily show that the soft set ξ_N is a SIN over U. Since N is a zero-symmetric NR, then $(\xi_N)_0 = \xi_N$ by Theorem 3.1. It is seen that

$$\xi_N(2) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

and

$$(\xi_N \circ \xi_N)(2) = \left\{ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \right\}.$$

Therefore, $\xi_N \not\subseteq \xi_N \circ \xi_N$.

As is seen, $\xi_M \circ \xi_M$ needs not be equal to ξ_M, however we have the following :

Theorem 5.2. *Let* ξ_N *be a* SIN *and* $(\xi_N)_0 = \xi_M$. *Then,*

$$(\xi_M \circ \xi_M)(0) = \xi_M(0).$$

Proof. $(\xi_M \circ \xi_M)(0) \subseteq \xi_M(0)$ is obvious by Theorem 5.1. Thus, we need to show that

$$(\xi_M \circ \xi_M)(0) \supseteq \xi_M(0).$$

Note that

$$(\xi_M \circ \xi_M)(0) \neq \emptyset, \quad since \;\; 0 = 0 \cdot 0.$$

So, let $x, y \in M$ such that $0 = xy$. Thus,

$$\begin{aligned} (\xi_M \circ \xi_M)(0) &= \bigcup_{0=xy} \{\xi_M(x) \cap \xi_M(y)\} \\ &\supseteq \xi_M(0) \cap \xi_M(0) \;\; (since \;\; 0 = 0 \cdot 0) \\ &= \xi_M(0) \end{aligned}$$

Thus, the proof is completed. □

It is known that if ξ_N is a SIN, then $\xi_N(0) \supseteq \xi_N(x)$ for all $x \in N$. Moreover, we have the following:

Theorem 5.3. *Let* $N = N_0$, ξ_N *be a* SIN *and* $(\xi_N)_0 = \xi_M$. *Then,*

$$(\xi_M \circ \xi_M)(0) \subsetneq \xi_M(x)$$

for all $x \in M$.

Proof. Let $x, y \in M$ such that $0 = xy$. Thus,

$$\begin{aligned} (\xi_M \circ \xi_M)(0) &= \bigcup_{0=xy} \{\xi_M(x) \cap \xi_M(y)\} \\ &\supseteq \xi_M(x) \cap \xi_M(0) \;\; (since \;\; N = N_0, \;\; 0 = x0) \\ &\supseteq \xi_M(x) \cap \xi_M(x) \;\; (by\ Lemma\ 2.1) \\ &= \xi_M(x) \end{aligned}$$

Thus, the proof is completed. □

It is known that $N_c N_c \subseteq N_c$. However, we have the following theorem:

Theorem 5.4. *Let* $N = N_c$, ξ_N *be a* SIN *and* $(\xi_N)_c = \xi_C$. *Then,*

$$\xi_C \circ \xi_C = \xi_C.$$

Proof. Here, first note that

$$(\xi_C \circ \xi_C) \neq \emptyset$$

since $N = N_c$, for all $c \in N$, $c = c \cdot 0$. So, let $x, y \in C$ such that $c = xy$. It follows that,

$$\begin{aligned}(\xi_C \circ \xi_C)(c) &= \bigcup_{c=xy} \{\xi_C(x) \cap \xi_C(y)\} \\ &\subseteq \bigcup_{c=xy} \xi_C(xy) \ \ (since\ \xi_C\ is\ a\ soft\ int\ \ near-ring) \\ &= \bigcup \xi_C(c) \\ &= \xi_C(c)\end{aligned}$$

Thus,

$$\xi_C \circ \xi_C \subset \xi_C.$$

Moreover,

$$\begin{aligned}(\xi_C \circ \xi_C)(c) &= \bigcup_{c=xy} \{f_C(x) \cap f_C(y)\} \\ &\supseteq f_C(c) \cap f_C(0) \ \ (since\ \ N = N_c,\ \ c = c0) \\ &\supseteq f_C(c) \cap f_C(c) \ \ (by\ \ Lemma\ \ 2.1) \\ &= f_C(c)\end{aligned}$$

Thus,

$$\xi_C \circ \xi_C \supset \xi_C \ \ and\ \ so\ \ \xi_C \circ \xi_C = \xi_C.$$

□

6. Soft uni-product applied on soft parts of soft uni near-rings

Definition 6.1. Let N be a NR and ξ_N and ζ_N be soft sets over the common universe U. Then, *soft uni-product* $\xi_N * \zeta_N$ is defined by

$$(\xi_N * \zeta_N)(x) = \begin{cases} \bigcap_{x=yz}\{\xi_N(y) \cup \zeta_N(z)\}, & \text{if } \exists y,z \in N \ \ such\ \ that\ \ x = yz, \\ \emptyset, & otherwise \end{cases}$$

for all $x \in N$. It is obvious that if N is a NR with identity, then the second condition does not exist.

Theorem 6.1. *Let N be a NR with identity, ξ_N be a SUN and $(\xi_N)_0 = \xi_M$. Then,*

$$\xi_M * \xi_M \supset \xi_M.$$

Proof. Since N is a NR with identity, there exist elements $x, y \in M$ such that $m = xy$. Thus,

$$\begin{aligned}(\xi_M * \xi_M)(m) &= \bigcap_{m=xy} \{\xi_M(x) \cup \xi_M(y)\} \\ &\supseteq \bigcap_{m=xy} \xi_M(xy) \\ &= \bigcap \xi_M(m) \\ &= \xi_M(m)\end{aligned}$$

Thus,

$$\xi_M * \xi_M \supset \xi_M.$$

Here, since ξ_N is a SUN, $\xi_N(xy) \subseteq \xi_N(x) \cup \xi_N(y)$ for all $x \in N$. And, since ξ_M is the restricted function of f to M, then $\xi_M(xy) \subseteq \xi_M(x) \cup \xi_M(y)$. That is, ξ_M is a SUN, too. □

Note that the converse, that is $\xi_M \supset \xi_M * \xi_M$ does not hold as seen in the following example.

Example 6.1. *Let $N = \{0, 1, 2, 3\}$ be the the (right) NR in Example 5.1. Assume that N is the set of parameters and $U = S_3$, symmetric group, is the universal set. We define a soft set ξ_N over U by*

$$\xi_N(0) = \{(1)\},\ \xi_N(1) = \{(1), (12), (13)\},\ \xi_N(2) = \{(1), (12)\},\ \xi_N(3) = \{(1), (12), (13))\}$$

Then, one can easily show that the soft set ξ_N is a SUN over U and $(\xi_N)_0 = \xi_N$ by Theorem 3.1. Moreover, all x element of N, can be expressed as $x = yz$. Hence,

$$\xi_N * \xi_N \neq \emptyset.$$

However, it is seen that

$$(\xi_N * \xi_N)(2) = \{(1), (12), (13)\},$$

thus, $\xi_N \not\supset \xi_N * \xi_N$.

As is seen, $\xi_M * \xi_M$ needs not be equal to ξ_M, however we have the following :

Theorem 6.2. *Let ξ_N be a SUN and $(\xi_N)_0 = \xi_M$. Then,*

$$(\xi_M * \xi_M)(0) = \xi_M(0).$$

Proof. First note that, since $0 = 0 \cdot 0$, $(\xi_M * \xi_M)(0) \neq \emptyset$. Moreover,

$$(\xi_M * \xi_M)(0) \supseteq \xi_M(0)$$

is obvious by Theorem 6.1. Therefore, we need to show that

$$(\xi_M * \xi_M)(0) \subseteq \xi_M(0).$$

Since

$$\begin{aligned}(\xi_M * \xi_M)(0) &= \bigcap_{0=xy} \{\xi_M(x) \cup \xi_M(y)\} \\ &\subseteq \xi_M(0) \cup \xi_M(0) \\ &= \xi_M(0)\end{aligned}$$

the proof is completed. □

It is known that if ξ_N is a SUN, then $\xi_N(0) \subseteq \xi_N(x)$ for all $x \in N$. Moreover, we have the following:

Theorem 6.3. *Let* $N = N_0$, ξ_N *be a* SUN *and* $(\xi_N)_0 = \xi_M$. *Then,*

$$(\xi_M * \xi_M)(0) \subseteq \xi_M(x)$$

for all $x \in M$.

Proof. Let $x, y \in M$ such that $0 = xy$. Thus,

$$\begin{aligned}(\xi_M * \xi_M)(0) &= \bigcap_{0=xy} \{\xi_M(x) \cup \xi_M(y)\} \\ &\subseteq \xi_M(x) \cup \xi_M(0) \ \ (since\ 0 = x0) \\ &\subseteq \xi_M(x) \cup \xi_M(x) \ \ (by\ \ Lemma\ \ 2.2) \\ &= \xi_M(x)\end{aligned}$$

Thus, the proof is completed. □

Theorem 6.4. *Let* N *be a* NR, $N = N_c$ *and* ξ_N *be a* SUN. *If* $(\xi_N)_c = f_C$, *then*

$$\xi_C * \xi_C = \xi_C.$$

Proof. Note that since $N = N_c$,

$$(\xi_C * \xi_C) \neq \emptyset.$$

Let $x, y \in C$ such that $c = xy$. It follows that,

$$\begin{aligned}(\xi_C * \xi_C)(c) &= \bigcap_{c=xy} \{f_C(x) \cup f_C(y)\} \\ &\supseteq \bigcap_{c=xy} f_C(xy) \\ &= \bigcap f_C(c) \\ &= f_C(c)\end{aligned}$$

Thus,

$$\xi_C * \xi_C \supset \xi_C.$$

Moreover,

$$\begin{aligned}(\xi_C * \xi_C)(c) &= \bigcap_{c=xy} \{\xi_C(x) \cup \xi_C(y)\} \\ &\subseteq \xi_C(c) \cup \xi_C(0) \ \ (since\ \ N = N_c,\ \ c = c0) \\ &\subseteq \xi_C(c) \cap \xi_C(c) \ \ \ (by\ \ Lemma\ \ 2.2) \\ &= \xi_C(c)\end{aligned}$$

Thus,

$$\xi_C * \xi_C \subset \xi_C \ \ and\ \ therefore,\ \ \xi_C * \xi_C = \xi_C.$$

□

7. Conclusion

The concepts soft int near-rings and soft uni near-rings were first introduced and studied in [52] and [53]. In this paper, by using the soft sets, we have defined soft zero-symmetric part and soft constant part of soft int near-rings and soft uni near-rings. We have obtained many results on some operations of soft sets which preserve under soft zero-symmetric part and soft constant parts. Furthermore, we have defined soft int-product and soft uni-product of soft int near-ring and soft uni near-ring and have given some applications of them to soft parts of soft int near-rings and soft uni near-rings, respectively. The construction of the soft 0-symmetric part and the soft constant part aids in the advancement of the soft near-ring theory as the 0-symmetric and constant sections of a near-ring structure are crucial for characterizing the near-ring. To extend this study, one can further study the relations between zero-symmetric part (resp. constant part) of a near-ring and soft zero-symmetric part (resp. soft constant part) of a soft int(or uni) near-ring.

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