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Submaximality under mIαg-closed sets in ideal minimal spaces

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Abstract: This article dealt a new submaximal space called mIαg-submaximal space in ideal minimal space. Significant properties of mIαg-submaximal space are studied. Equivalent conditions concerned with mIαg-submaximal space and mIαg-locally m^*-closed sets, $m*$-codense sets, pre-m-I-open sets are also established

Key words: mIαg-closed sets, mIαg-locally m^*-closed sets, mIαg-submaximal spaces

1. Introduction

Submaximality in general topological spaces was introduced by Hewit [7]. He defined a general way of constructing maximal topologies. A systemaized approach on submaximality in topological spaces is followed by Arhangel'skii et.al [1]. Necessary and sufficient conditions of submaximality are also proved by them. They have also proved that the submaximal space is left separated in a topological space. The concept, ideal in topological spaces was studied by Kuratowski [9] and Vaidyanathaswamy [16]. Several properties of ideal topological spaces were discussed by Jankovic et.al [8]. Properties of I-submaximal spaces was studied by Erdal Ekici et.al [6]. Ig-submaximal spaces was introduced by Bhavani et.al [3]. The concept of minimal structure and minimal spaces were introduced and studied by maki et.al [10]. Quite recently [11] Ozbakir et.al studied the impact of ideals in minimal spaces and termed as ideal minimal spaces. They have initiated A_m^*, called the minimal local function in ideal minimal spaces. α-generalised closed sets (briefly mIαg-closed sets) in ideal minimal spaces was introduced and some significant properties were studied by Parimala et.al [2]. The concept Locally closed, on mIαg closed sets (briefly mIαglocally m^*-closed sets) was defined by Parimala et.al [12] In this paper we have proved some equivalent conditions of mIαg locally m^*-closed sets. Further we have defined mIαg-submaximal spaces in ideal minimal spaces and discussed significant properties of and mIαg-submaximal spaces. Also we have proved several equivalent conditions on mIαg-submaximal spaces.

2. Preliminaries

Definition 2.1. [9] Let X be a set and is non empty. Let I be the collection of subsets of X which is also non empty. I is referred as an ideal if it satisfies the conditions. Let A, B be any two subsets of I such that (i) $A \in I$ and $B \subset A$ implies $B \in I$ (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

Definition 2.2. [10] Consider a set X and M represents the set of all possible subsets of X. M is termed as the minimal structure if ϕ and X should be the members of M. The minimal spaces we mean the set X with the minimal structure M say (X, M). The elements of M are referred as m-open sets and their complements

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are called m-closed sets. The interior and closure of m-open sets are denoted by m-int and m-cl respectively and are defined as m-$int(A)=\cup\{U : U \subset A, U \in M\}$, m-$cl(A)= \cap\{F : A \subset F, X - F \in M\}$.

Remark 2.3.[11] The minimal space (X, M) is said to exhibit the property $[U]$ if the union of any number of m-open sets is a m-open set and the property $[I]$ if the intersection of finite number of m-open sets is a m-open set.

Definition 2.4. [11] Let $P(X)$ denotes the power set of (X, M, I). The mapping $(.)^*_m : P(X) \longrightarrow P(X)$ leads the definition of the minimal local function A^*_m as $\{x \in X : U_m \cap A \notin I\}$ for all $U_m \in U_m(x)$.

Theorem 2.5. [11] In a minimal space (X, M), let I_1, I_2 be two ideals on X. K_1 and K_2 be subsets of X. Then

(a) $K_1 \subset K_2 \Rightarrow K_1m^* \subset K_2m^*$.

(b) $K_1m^* \cup K_2m^* \subset (K_1 \cup K_2)^*_m$.

(c) $(K^*_m)^*_m \subset K^*_m$.

(d) $K^*_m = m$-$cl(K^*_m) \subset m$-$cl(K)$.

(e) $I_1 \subset I_2 \Rightarrow K^*_m(I_2) \subset K^*_m(I_1)$.

Remark 2.6. [11] If the ideal minimal space (X, M) includes the property $[I]$, then (b) of Theorem 2.5 satisfies the equality. That is $K_1m^* \cup K_2m^* = (K_1 \cup K_2)^*_m$.

Definition 2.7. [11] The minimal $*$-closure operator m-cl^* on a subset A of (X, M, I) is defined as the union of A and A^*_m. That is m-$cl^*(A) = A \cup A^*_m$. The minimal structure on m-cl^* is termed as $M^*(I, M)$ which is defined as $M^*(I, M) = \{F \subset X : m\text{-}cl^*(X - F) = X - F\}$. The members of M^* are named as m^*-open sets. The interior of m^*-open sets is denoted by m-$int^*(A)$.

Proposition 2.8.[11] Salient features of the minimal $*$ -closure operator m-cl^* are listed below. Let $K, K_1, K_2 \subset X$

(a) m-$cl^*(K_1) \cup m$-$cl^*(K_2) \subset m$-$cl^*(K_1 \cup K_2)$.

(b) If $K_1 \subset K_2$, then m-$cl^*(K_1) \subset m$-$cl^*(K_2)$.

(c) $K \subset m$-$cl^*(K)$.

(d) m-$cl^*(\phi) = \phi$ and m-$cl^*(X) = X$.

Remark 2.9. [11] When the ideal minimal space (X, M, I) includes the property $[I]$ then equality holds in (a) of Theorem 2.8. That is m-$cl^*(K_1 \cup K_2) = m$-$cl^*(K_1) \cup m$-$cl^*(K_2)$ and also m-$cl^*(m$-$cl^*(K)) = m$-$cl^*(K)$ for the subset K.

Definition 2.10. [11] In an ideal minimal space (X, M, I), a subset A is termed as m^*-dense in itself set if A is a subset of A^*_m (briefly $A \subset A^*_m$).

Lemma 2.11.[11] If a subset A is seems to be a m^* dense in itself set in an ideal minimal space, then the following equality holds. $A^*_m = m$-$cl(A^*_m) = m$-$cl(A) = m$-$cl^*(A)$.

Definition 2.12 [4] A subset A of X is defined as a m^*-dense set if m-$cl^*(A) = X$.

Definition 2.13. [2] Let A be non empty subset of X, then A is defined to be a mIαg-closed set if $A^*_m \subseteq U$ whenever $A \subseteq U$, U is a αm-open set.

Lemma 2.14. [2] If a set A is a m^*-closed set, then it is be a mIαg-closed set.

Definition 2.15. Consider a subset A of X, then A is defined to be a

(1) I-locally m^*-closed set [14] if A equals the intersection of a m-open set U and a m^*-closed set F. That is $A = U \cap F$.

(2) mIαg-locally m^*-closed set [12] if there exists a mIαg-open set U and a m^*-closed set F such that $A = U \cap F$.

(3) pre-mI-open set [13] if $A \subset m\text{-}int(m\text{-}cl^*(A))$.

Lemma 2.16. [4] m^*-dense sets are pre-mI-open sets.

Definition 2.17. [4] A subset A of a minimal space X is termed to be

(a) m-I-dense if $A^*_m = X$.

(b) m^*-codense set if $(X - A)$ is a m^*-dense set.

Definition 2.18. The ideal minimal space (X, M, I) is defined to be a

(a) m-T_1-space [15] if there exists two elements x, y of X such that $x \cap y = \phi$, there exists a m-open set P that contains x, but not y and another m-open set Q that contains y, but not x.

(b) m-I-submaximal space [5] if every m^*-dense subset of X is a m-open set.

3. mIαg-locally m^*-closed set

Some Characterisations of mIαg-locally m^*-closed set are as follows.

Theorem 3.1. The necessary and sufficient conditions of a subset A to be a mIαg-closed set is $m\text{-}cl^*(A) \subseteq U$, $A \subset U$ where U is a m-open set.

Proof. Necessity. Consider a mIαg-closed set A and a m-open set U. Since $m\text{-}cl^*(A) = A \cup A^*_m$. Also we have $A \subseteq U$, $A^*_m \subset U$ and U is a m-open set. Therefore $m\text{-}cl^*(A) \subseteq U$.

Proof. Sufficiency. Let $m\text{-}cl^*(A) \subseteq U$, $A \subseteq U$ and U is a m-open set. Since $m\text{-}cl^*(A) = A \cup A^*_m \subseteq U$, we get $A, A^*_m \subseteq U$, U is m-open. Since every m-open set is a αm-opens set, $A^*_m \subseteq U$, $A \subseteq U$, U is a αm-open set. Therefore, A is a mIαg-closed set.

Theorem 3.2. A m-closed set is always a mIαg closed set, equivalently (A m-open set is a mIαg-open set.)

Proof. Consider a αm open set U and let $A \subseteq U$ be a m-closed set in X, then $m\text{-}cl(A) = A$, which implies $m\text{-}cl(A) \subseteq U$. It is clear that $m\text{-}cl^*(A) \subseteq m\text{-}cl(A) \subset U$. Since $m\text{-}cl^*(A) = A \cup A^*_m$ and $A \subseteq U$ we get $A^*_m \subseteq U$, where U is a αm-open set. Hence A is a mIαg closed set.

Theorem 3.3. If a subset A of (X, M, I) is a I-locally m^*-closed set, then it is a mIαg-locally m^*-closed set and the converse of this theorem may not be true explained in Example 3.4.

Proof. Referring Theorem 3.2, the proof follows from the definition of I-locally m^*-closed set.

Example 3.4. (X, M, I) be a ideal minimal space with $X = \{a, b, c\}$, $M = \{\phi, X, \{a\}, \{b, c\}\}$ and $I = \{\phi, \{c\}\}$. In this example mIαg-locally m^*-closed sets are the elements of the power set of X, but the $\{b\}$ is not a locally m^*-closed set.

Theorem 3.5. Consider the ideal minimal space (X, M, I) and $A \subset X$ is m^*-dense in itself then statements below given are equivalent.

(a) A is an mIαg-locally m^*-closed set.

(b) A equals the intersection of a mIαg-open set U and m-$cl^*(A)$. That is $A = U \cap m\text{-}cl^*(A)$ for a mIαg-open set U.

(c) m-$cl^*(A) - A$ and $A^*_m - A$ are equal and also mIαg closed sets.

(d) $A \cup (X - m\text{-}cl^*(A))$ and $A \cup (X - A^*_m)$ are equal and also mIαg-open sets.

(e) $A \subset m\text{-}int(A \cup (X - A^*_m))$

Proof. $(a) \Longrightarrow (b)$. If A is an mIαg-locally m^*-closed set, then $A = U \cap F$ where U is mIαg-open and F is m^*-closed, that is $F^*_m \subset F$. Since $A = U \cap F$, we have $A \subset U$ and $A \subset m\text{-}cl^*(A)$. Therefore $A \subset U \cap m\text{-}cl^*(A)$. Since F is m^*-closed and $A \subset F$, $m\text{-}cl^*(A) \subset m\text{-}cl^*(F)$, which implies $U \cap m\text{-}cl^*(A) \subset U \cap m\text{-}cl^*(F) \subset U \cap F = A$. Then $A = U \cap m\text{-}cl^*(A)$.
$(b) \Longrightarrow (c)$. Consider $m\text{-}cl^*(A) - A = A^*_m - A = A^*_m \cap (X - A) = A^*_m \cap (X - (U \cap m\text{-}cl^*(A))) = A^*_m \cap (X - U)$. Let V be a m-open set such that $m\text{-}cl^*(A) - A \subset V$. Then $A^*_m \cap (X - U) \subset V$, which implies $(X - U) \subset ((X - A^*_m) \cup V)$. Since U is mIαg-open set $(X - U)$ is a mIαg-closed set. Therefore by Theorem 3.1, we get $m\text{-}cl^*(X - U) \subset ((X - A^*_m) \cup U$. Also $A^*_m \cap m\text{-}cl^*(X - U) \subset V$. Consider $A^*_m \cap (X - U) \subset A^*_m$ and $A^*_m \cap (X - U) \subset (X - U)$. By Theorem 2.5(a)we get $(A^*_m \cap (X - U))^*_m \subset (A^*_m)^*_m \subset A^*_m$ and $(A^*_m \cap (X - U))^*_m \subset (X - U)^*_m \subset m\text{-}cl^*(X - U)$. Therefore $(A^*_m \cap (X - U))^*_m \subset A^*_m \cap m\text{-}cl^*(X - U) \subset V$. Also $m\text{-}cl^*(A) - A = A^*_m \cap (X - U)$. Therefore we get $(m\text{-}cl^*(A) - A)^*_m \subset A^*_m \cap m\text{-}cl^*(X - U) \subset V$. That is $(m\text{-}cl^*(A) - A)^*_m \subset V$ and V is a m-open set and hence V is a αm-open set. Finally we have proved that $(m\text{-}cl^*(A) - A)^*_m \subset V$ whenever $(m\text{-}cl^*(A) - A) \subset V$, V is a αm-open set, which implies $(m\text{-}cl^*(A) - A) = A^*_m - A$ is a mIαg-closed set.
$(c) \Longrightarrow (d)$ Since $m\text{-}cl^*(A) - A$ is mIαg-closed, $(X - (m\text{-}cl^*(A) - A))$ is mIαg-open, which implies $A \cup (X - m\text{-}cl^*(A)) \Rightarrow A \cup (X - (A \cup A^*_m))$ is mIαg-open and hence $A \cup (X - A^*_m)$ is mIαg-open.
$(d) \Longrightarrow (e)$ Since $A \subset (A \cup (X - A^*_m))$, $m\text{-}intA \subset m\text{-}int(A \cup (X - A^*_m))$. Therefore $A \subset m\text{-}int(A) \subset m\text{-}int(A \cup (X - A^*_m))$ and hence $A \subset m\text{-}int(A \cup (X - A^*_m))$.
$(e) \Longrightarrow (a)$ By (e) we can say $A \cup (X - A^*_m) \subset m\text{-}int(A \cup (X - A^*_m))$. By (d) $A \cup (X - m\text{-}cl^*(A)) = A \cup (X - A^*_m)$ is a mIαg-open set. Also $A \cup (X - m\text{-}cl^*(A)) \cap m\text{-}cl^*(A) = (A \cap m\text{-}cl^*(A)) \cup ((X - m\text{-}cl^*(A)) \cap m\text{-}cl^*(A))) = (A \cap (A \cup A^*_m)) \cup \phi = A \cup (A \cap A^*_m) \cup \phi = A$ since A is a m^* dense set. Also since A is a m^* dense set, $A \subset A^*_m)$. Therefore $m\text{-}cl^*(A = A^*_m))$. Hence $(m\text{-}cl^*(A))^*_m = (A^*_m)^*_m = A^*_m = m\text{-}cl^*(A)$. That is $(m\text{-}cl^*(A))^*_m \subset m\text{-}cl^*(A)$. Therefore $m\text{-}cl^*(A)$ is a m^*-closed set. That is $A = (A \cup (X - A^*_m) \cap m\text{-}cl^*(A)$ such that $A = (A \cup (X - A^*_m)$ is a mIαg-open set and $m\text{-}cl^*(A)$ is a m^*-closed set. Therefore A is mIαg-locally m^*-closed set.
Theorem 3.6. If G is a m-open subset of the ideal minimal space (X, M, I), then G is mIαg-locally m^*-closed set, but the converse need not be true.
Proof. Lemma 2.14 and Theorem 3.2 proves the Theorem.
Example 3.7. Let the ideal minimal space with $X = \{a, b, c\}$, $M = \{\phi, X, \{a\}, \{b, c\}\}$ and $I = \{\phi, \{c\}\}$. In this example mIαg-locally m^*-closed sets are the elements of the power set of X, but the $\{b\}$ is not a m-open set.

Theorem 3.8. A mIαg-locally m^*-closed set which is also a m-I-dense set, is a mIαg-open set.

Proof. Referring Theorem 3.5(d) if a set A is a mIαg-locally m^*-closed set then $A \cup (X - m\text{-}cl^*(A))$ is a mIαg-open set. By the definition of m-I-dense set we get $A^*_m = X$. Therefore $m\text{-}cl^*(A) = A \cup A^*_m = X$, which implies $A \cup (X - X) = A$ is a mIαg-open set.

Corollary 3.9. Let A be an m-I-dense subset of X, then A is a mIαg-locally m^*-closed set if and only of A is a mIαg-open set.

Theorem 3.10. Let (X, M, I) be an ideal minimal space satisfying property $[I]$, then we can prove the equivalent statements on mIαg-locally m^*-closed sets.

(a) Every subset of X is a mIαg-locally m^*-closed set.

(b) Every m^*-dense set a mIαg-open set.

Proof. $(a) \Longrightarrow (b)$ The proof follows from Theorem 3.5(d).
$(b) \Longrightarrow (a)$ Consider a subset K of X. Let $S = K \cup (X - m\text{-}cl^*(K))$. Then $m\text{-}cl^*(S) = m\text{-}cl^*(K \cup (X - m\text{-}cl^*(K))) = (K \cup (X - m\text{-}cl^*(K)) \cup (K \cup (X - m\text{-}cl^*(K)))^*_m = (K \cup (X - m\text{-}cl^*(K))) \cup K^*_m \cup (X - m\text{-}cl^*(K))^*_m = m\text{-}cl^*(K) \cup m\text{-}cl^*(X - m\text{-}cl^*(K)) = m\text{-}cl^*(K) \cup (X - m\text{-}cl^*(K)) = X$. That is we have proved $m\text{-}cl^*(S) = X$. Therefore by Definition 2.12 S is a m^*-dense set. By (b) S is a mIαg-open set. Therefore by Theorem 3.5(d), K is a mIαg-locally m^*-closed set. **Theorem 3.11.** Let (X, M, I) be an ideal minimal space satisfying the property $[U]$ and $A \subset X$, is a m^*-dense set. The necessary and sufficient condition that the set A is a mIαg-locally m^*-closed set is $A \cap (A^*_m - A)^*_m$ is a mIαg-open set.

Proof. Let $A \subset X$. Consider $(m\text{-}cl^*(A) \cap (X - A))^*_m - (m\text{-}cl^*(A) \cap (X - A)) = (m\text{-}cl^*(A) \cap (X - A))^*_m \cap (X - (m\text{-}cl^*(A) \cap (X - A)) = m\text{-}cl^*(A) \cap (X - A))^*_m \cap ((X - m\text{-}cl^*(A)) \cup A) = (m\text{-}cl^*(A) \cap (X - A))^*_m \cap (X - m\text{-}cl^*(A)) \cup (m\text{-}cl^*(A) \cap (X - A)^*_m \cap A) = \phi \cup (m\text{-}cl^*(A) - A)^*_m \cap A = (A^*_m - A)^*_m \cap A$ by Theorem 3.5(c) A is a mIαg-locally m^*-closed set implies $m\text{-}cl^*(A) - A = m\text{-}cl^*(A) \cap (X - A)$ is a mIαg-closed set. Hence $(m\text{-}cl^*(A) \cap (X - A))^*_m - (m\text{-}cl^*(A) \cap (X - A))$ is a mIαg-open set. That is $(A^*_m - A)^*_m \cap A$ is a mIαg-open set.

4. mIαg-Submaximal spaces

Definition 4.1. An ideal minimal space (X, M, I) is said to be a mIαg-submaximal space if every m^*-dense subset in X is a mIαg-open set.

Theorem 4.2. Every mI-submaximal space is a mIαg-submaximal space, but the converse of this statement may not be true.

Proof. Consider a m^*-dense set A of a mI-submaximal space X. By the definition of mI-submaximal space A is a m-open set. By Theorem 3.2, A is a mIαg-open set in X. Therefore (X, M, I) is a mIαg-submaximal space.

Example 4.3. Consider the ideal minimal space (X, M, I) with $X = \{a, b, c, d\}$ and $M = \{\phi, X, \{a, b\}, \{b, c\}, \{a, c, d\}\}$ and the ideal $I = \{\phi, \{c\}\}$. m^*-dense sets are $\{\{a, b\}, \{a, b, c\}, \{a, b, c\}\}$. In this example (X, M, I) is a mIαg-submaximal space, but not a mI-submaximal space since $\{a, b, d\}$ is a m^*-dense set, but not an m-open set.

Lemma 4.4. When (X, M, I) represents a ideal minimal space and $A \subset M$, then below said statements are equivalent.

(a) A is a mIαg-closed set.

(b) For a m-open set U of X, m-$cl^*(A) \subset U, A \subset U$.

(c) Let $x \in m$-$cl^*(A)$, m-$cl(x) \cap A$ is non empty.

(d) m-$cl^*(A) - A$ does not have non empty m-closed sets.

(e) $A^*_m - A$ does not have non empty m-closed sets.

Theorem 4.5. If (X, M, I) is a m-T_1 ideal space and A is a m^*-dense in itself set and mIαg-closed subset of X, then A is a m-closed set.

Proof. Let A be m^*-dense in itself set and mIαg-closed subset of X. Then by Lemma 4.4 there exists no non empty m-closed set in m-$cl^*(A) - A$. Also since X is a m-T_1, A is a m^*-closed set we have m-$cl^*(A) - A = \phi$. A is a m^*-closed set implies m-$cl^*(A) = m$-$cl(A)$. Therefore m-$cl(A) - A = \phi$ which implies m-$cl(A) = A$. Hence A is a m-closed set.

Lemma 4.6 In a ideal minimal space (X, M, I) if a subset A is a pre-m-I-open set, then A can be expressed as $A = G \cap B$, where G is a m-open set and B is a m^*-dense set.

Proof. Consider a pre-m-I-open set A, we have $A \subseteq m$-$int(m$-$cl^*(A) = U$ where U is a m-open set. Let $D = X - (U - A) = X - (U \cap A^c) = X \cap (U \cap A^c)^c = X \cap (U^c \cup A) = (U^c \cup A) = (X - U) \cup A$. To prove D is a m^*-dense set it is enough to prove m-$cl^*(D) = X$. That is m-$cl^*((X - U) \cup A) \supseteq m$-$cl^*(X - U) \cup m$-$cl^*(A) \supseteq (X - m$-$cl^*(A) \cup m$-$cl^*(A) = X$. Therefore D is m^*-dense set. Also $U \cap D = U \cap ((X - U) \cup A) = (U \cap (X - U)) \cup (U \cap A) = \phi \cup (U \cap A) = \phi \cup A = A$ here $A \subseteq U$. That is $A = U \cap D$ such that U is a m-open set and D is a m^*-dense set.

Remark 4.7. Property $[\jmath]$ refers "union of two mIαg-closed sets is a mIαg-closed set" and property $[\upsilon]$ refers "For any two mIαg-closed sets A and B, $A \cap B$ is also a mIαg-closed set".

Theorem 4.8. Let (X, M, I) be the ideal minimal space satisfying property $[\upsilon]$ then statements below which are equivalent.

(a) (X, M, I) is a mIαg-submaximal space.

(b) If A is a pre-m-I-open set, then A is a mIαg-open set.

Proof. $(a) \Longrightarrow (b)$. Let (X, M, I) is a mIαg-submaximal space and $A \subset X$ be a pre-m-I-open set. By Lemma 4.6 $A = U \cap D$, U is a m-open set and D is a m^*-dense set. Since X is a mIαg-submaximal space and D is a mIαg-open set in X and U is a m-open set, implies U is a mIαg-open by Theorem 3.2, therefore A is a mIαg-open set.

$(b) \Longrightarrow (a)$. Let A be m^*-dense subset of X, then by Lemma 2.16 A is a pre-mI-open set. By hypothesis A is a mIαg-open set. Therefore X is a mIαg-submaximal space.

Theorem 4.9. Let (X, M, I) be the ideal minimal space satisfying property $[\upsilon]$ then the following statements are equivalent.

(a) (X, M, I) is a mIαg-submaximal space.

(b) If A is a subset of X, then A is a mIαg-locally m^*-closed set.

(c) Any m^*-dense subset of X is the intersection of a m^*-closed set and a mIαg-open subset of X.

Proof. $(a) \Longrightarrow (b)$. Let (X, M, I) be a mIαg-submaximal space, by definition of mIαg-submaximal space every m^*-dense set is a mIαg-open set. Referring Theorem 3.10, every subset is a mIαg-locally m^*-closed set.
$(b) \Longrightarrow (c)$. Let A be a m^*-dense set, by (a) of this theorem A is mIαg-locally m^*-closed set. Theorem 3.5(b) explains, there exists a mIαg-open set U such that $A = U \cap m\text{-}cl^*(A)$. Consideration of A as a m^*-dense set we inferred that $m\text{-}cl^*(A) = X$. Hence $A = U \cap m\text{-}cl^*(A) = U \cap X = U$ and U is a mIαg-open set in X.
$(c) \Longrightarrow (a)$. Let A be m^*-dense set in (X, M, I), by (c) $A = U \cap E$, U is a mIαg-open set and E is a m^*-closed set. Since $A \subset E$, E is m^*-dense set. That is $m\text{-}cl^*(E) = X$. Since E is a m^*-closed set $m\text{-}cl^*(E) = E = X$. Hence $A = U \cap E = U \cap X = U$ and it is a mIαg-open set. Therefore (X, M, I) is mIαg-submaximal space.
Theorem 4.10. For an ideal minimal space, the following statements are equivalent.

(a) (X, M, I) is a mIαg-submaximal space.

(b) Whenever the subset A is not a mIαg-open set, we can prove $A - (m\text{-}int(m\text{-}cl^*(A))) \neq \phi$.

Proof. $(a) \Longrightarrow (b)$. Assume the contrary that $A - (m\text{-}int(m\text{-}cl^*(A))) = \phi$. Therefore $A \subset (m\text{-}int(m\text{-}cl^*(A)))$, means A is a pre-mI-open set. Since X is a mIαg-submaximal space, by Theorem 4.8, A is mIαg-open. Which is a contradiction to our assumption that A is not a mIαg-open set. Therefore $A - (m\text{-}int(m\text{-}cl^*(A)))$ is non empty.
$(b) \Longrightarrow (a)$. Consider a pre-mI-open set A which is not a mIαg-open set, then by (b) $A - (m\text{-}int(m\text{-}cl^*(A)))$ is non empty, which implies $A \nsubseteq (m\text{-}int(m\text{-}cl^*(A)))$. That is A may not be a pre-mI-open set. Which contradicts to our assumption that A is Pre-mI-open. Therefore A is a mIαg-open set and hence referring Theorem 4.8 X is mIαg-submaximal space.
Theorem 4.11. Consider an ideal minimal space (X, M, I) with the property $[v]$, then it is possible for the equivalent statements on mIαg-submaximal spaces.

(a) (X, M, I) is a mIαg-submaximal space.

(b) The family of all mIαg-open sets ζ such that $\zeta = \{U - A : U$ is mIαg-open and $m\text{-}int^*(A) = \phi\}$.

Proof. $(a) \Longrightarrow (b)$. Let X mIαg-submaximal space. Construct a σ as $\sigma = \{U - A : U$ is mIαg-open and $m\text{-}int^*(A) = \phi\}$. Our aim is to prove $\sigma = \zeta$. Consider an element $G \in \zeta$. Since $G = G - \phi$ and $m\text{-}int^*(\phi) = \phi$, $G \in \sigma$.
Hence $\zeta \subset \sigma$.———(1)
Let $G \in \sigma$. It is sufficient to prove G is a mIαg-open set. Since $G \in \sigma$, G can be written as $G = U - A$ such that U is a mIαg-open set and $m\text{-}int^*(A) = \phi$. Also $G = U - A = U \cap (X - A)$. Since $m\text{-}int^*(A) = \phi$, $X - (m\text{-}int^*(X - A)) = m\text{-}cl^*(A) = X$. That is $(X - A)$ is a m^*-dense set. Since X is a mIαg-submaximal space, $(X - A)$ is a m^*-dense set implies $(X - A)$ is a mIαg-open set. Hence $G = U \cap (X - A)$ is a mIαg-open set and so
$\sigma \subset \zeta$.————(2)
combining equations (1) and (2)we have proved $\sigma = \zeta$. Hence (b) can be followed.
$(b) \Longrightarrow (a)$. Consider A be a pre-mI-open set. With reference of Lemma 4.6. A can be written as the intersection of the sets G and B such that G is a m-open set and B is a m^*-dense set. Therefore $m\text{-}cl^*(B) = X$ and so $m\text{-}int^*(X - B) = \phi$. That is $A = G \cap B = G - (X - B)$ and $m\text{-}int^*(X - B) = \phi$. By Theorem 3.2 G is a mIαg-open set. Therefore $A \subset \zeta$ and hence A is a mIαg-open-set. That is assumption of

the pre-mI-open set A leads that A is mIαg-open. Hence Theorem 4.8. infers that X is a mIαg-submaximal space.

Theorem 4.12. Let (X, M, I) be an ideal minimal space satisfying the property $[I]$ then the equivalent statements on mIαg-submaximal spaces are as follows.

(a) X is a mIαg-submaximal space.

(b) There exists a mIαg-closed set $m\text{-}cl^*(A) - A$ for every subset A of X.

Proof. $(a) \Longrightarrow (b)$. Consider a mIαg-submaximal space (X, M, I) and let $A \subset X$. To prove $m\text{-}cl^*(A) - A$ is mIαg-closed set is sufficient to prove $X - (m\text{-}cl^*(A) - A)$ is mIαg-open. We need to prove $X - (m\text{-}cl^*(A) - A)$ is a m^*-dense set. It is necessary to prove that $m\text{-}cl^*(X - (m\text{-}cl^*(A) - A)) = X$. Proceeding as, $m\text{-}cl^*(A \cup (X - (m\text{-}cl^*(A)))) = m\text{-}cl^*(A) \cup m\text{-}cl^*(X - (m\text{-}cl^*(A))) = X$. Therefore $(X - (m\text{-}cl^*(A) - A))$ is a m^*-dense set. Also since X is a mIαg-submaximal space, by the definition of mIαg-submaximal space every m^*-dense set is a mIαg-open set. Hence $(X - (m\text{-}cl^*(A) - A)$ is a mIαg-open set. Therefore $(m\text{-}cl^*(A) - A)$ is a mIαg-closed set.

$(b) \Longrightarrow (a)$. Let $(m\text{-}cl^*(A) - A)$ be a mIαg-closed set and let $A \subset X$ be a m^*-dense set in (X, M, I). Therefore $m\text{-}cl^*(A) = X$, implies $(X - A$ is a mIαg-closed set. So A is mIαg-open. (X, M, I) is mIαg-submaximal space.

Theorem 4.13. Consider a m^*-dense subset A, which is also a m^*-dense in itself set in a ideal minimal space (X, M, I), then the equivalent statements are as follows.

(a) (X, M, I) is a mIαg-submaximal space.

(b) $A \cap (A^*_m - A)^*_m$ is a mIαg-open set for every $A \subset X$.

Proof. $(a) \Longrightarrow (b)$. Let (X, M, I) is a mIαg-submaximal space. By Theorem 4.9, every subset of A of X is a mIαg-locally m^*-closed set. Then by Theorem 3.11 $A \cap (A^*_m - A)^*_m$ is a mIαg-open set in X.

$(b) \Longrightarrow (a)$ By hypothesis $A \cap (A^*_m - A)^*_m$ is mIαg-open for every $A \subset X$. Since A is a m^*-dense set we have $m\text{-}cl^*(A) = X$. So that $A^*_m - A = m\text{-}cl^*(A) - A = X - A$. Hence $A \cap (A^*_m - A)^*_m = A \cap (X - A)^*_m = (X - A)^*_m - (X - A)$ is a mIαg-open set and hence $(X - A)$ is a mIαg-closed set, implies A is a mIαg-open set. Therefore (X, M, I) mIαg-submaximal space.

Theorem 4.14. For a topological space with minimal structure M and the ideal I, then the following statements are equivalent.

(a) (X, M, I) is a mIαg-submaximal space.

(b) If a subset A is a m^*-codense subset of X, then it is a mIαg-closed set.

Proof. $(a) \Longrightarrow (b)$. Let A be a m^*-codense set, then $(X - A)$is a m^*-dense set. By Theorem 4.13, $(X - A)$ is a mIαg-open set. Hence A is a mIαg-closed set.

$(b) \Longrightarrow (a)$. Since A is a m^*-codense set $(X - A)$ is a m^*-dense set and so $(X - A)$ is a mIαg-open set. Therefore by Theorem 4.13 (X, M, I) is a mIαg-submaximal space.

Theorem 4.15. Let (X, M, I) be an ideal minimal space, where I is a m-codense ideal.If every subset is m-I-locally -m-closed, then (X, M, I) is a $mI\alpha g$-submaximal space.

Proof. Let A be m^*-dense set. Since I is m-codense, A is ,m-I-dense and m-I-locally -m-closed,then A is

m-open and hence $mI\alpha g$-open. Therefore X is $mI\alpha g$-submaximal space.

Theorem 4.16. In a $mI\alpha g$-submaximal space (X, M, I) if for any two ideals I, I' such that $I \subset I'$, then (X, M, I') is a $mI'\alpha g$-submaximal space.

Proof. Let A be $m^*(I')$-dense subset in (X, M, I'), then $A \cup A^*_m(I') = X$. As $I \subset I'$ we get $A^*_m(I') \subset A^*_m(I)$, which implies that $X = A \cup A^*_m(I') \subset A \cup A^*_m(I)$. That is $A \cup A^*_m(I) = X$. Thus A is a $m^*(I)$-dense set. As (X, M, I) is $mI\alpha g$-submaximal, A is $mI\alpha g$-open. Since $I \subset I'$, A is $mI'\alpha g$-open. Therefore (X, M, I') is $mIi\alpha g$-submaximal space.

5. Conclusion

In this article we have discussed some salient features of $mI\alpha g$-locally m^*-closed sets. We have introduced a new submaximality called $mI\alpha g$-submaximality in ideal minimal spaces and studied its significant features. Equivalence of $mI\alpha g$-submaximality with Pre-mI-open sets, $mI\alpha g$-locally m^*-closed sets, m^*-codense sets are given. Heredity nature of ideals are imported under $mI\alpha g$-submaximality. Future work of this article will be in hyper connectedness and $mI\alpha g$-submaximality.

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