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Fuzzy Quotient-3 Cordial Labeling of Star Related Graphs - Paper II

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Abstract: Let G be a graph of order p and size q. Let $\sigma : V(G) \to [0,1]$ be a function defined by $\sigma(v) = \frac{r}{10}$, $r \in Z_4 - \{0\}$. For each edge uv define $\mu : E(G) \to [0,1]$ by $\mu(uv) = \frac{1}{10}\left\lceil \frac{3\sigma(u)}{\sigma(v)} \right\rceil$ where $\sigma(u) \leq \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $i \neq j$. The number of vertices having label i denotes $v_\sigma(i)$ and the number of edges having label i denotes $e_\mu(i)$ [3]. Here it is proved that Subdivision Star, Subdivision bistar, Splitting graph of star and bistar are Fuzzy Quotient-3 Cordial.

Key words: Subdivision Star, Subdivision bistar, Splitting graph, Fuzzy quotient-3 cordial graph.

1. Introduction

Graphs considered here are finite and simple. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. The cordial labeling concept was first introduced by cahit [2]. The quotient-3 cordial labeling have been introduced by Dr. P. Sumathi, A. Mahalakshmi and A. Rathi found in [3–7]. Motivated by these labelings we introduced fuzzy quotient-3 cordial labeling of graphs and proved some star related graphs are fuzzy quotient-3 cordial labeling [9].

Let $\sigma : V(G) \to [0,1]$ be a function defined by $\sigma(v) = \frac{r}{10}$, $r \in Z_4 - \{0\}$. For each edge uv define $\mu : E(G) \to [0,1]$ by $\mu(uv) = \frac{1}{10}\left\lceil \frac{3\sigma(u)}{\sigma(v)} \right\rceil$ where $\sigma(u) \leq \sigma(v)$. The function σ is called fuzzy quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with i and the number of edges labeled with j differ by at most 1 where $i,j \in \left\{\frac{r}{10}, r \in Z_4 - \{0\}\right\}$, $i \neq j$. The number of vertices having label i denotes $v_\sigma(i)$ and the number of edges having label i denotes $e_\mu(i)$ [9].

Definition 1.1. The graph obtained by attaching n number of pendant edges to the vertex is said to be a star graph and it is denoted by $G_{1,n}$.

Definition 1.2. The graph obtained by joining the two copies of star graph through an edge is called a bistar and is denoted by $G_{n,n}$.

Definition 1.3. The subdivision graph is obtained from the graph G by adding a new vertex between each pair of adjacent vertices of the graph G and it is denoted by $S(G)$.

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Definition 1.4. The graph obtained by adding a new vertex u' to each vertex u of G such that u is adjacent to every vertex that is adjacent to u in G is called the splitting graph of G and it is denoted by $S'(G)$.

2. Main Result

Theorem 2.1. *The star graph $S(G_{1,n})$ is fuzzy quotient-3 cordial.*

Proof. Let G be a $S(G_{1,n})$ graph.
Let $V(G) = \{x\} \cup \{x_k, y_k : 1 \leq k \leq n\}$ and $E(G) = \{(xx_k) : 1 \leq k \leq n\} \cup \{(x_k y_k) : 1 \leq k \leq n\}$.
Here $|V(G)| = 2n + 1$, $|E(G)| = 2n$.
Define $\sigma : V(G) \to [0, 1]$ as follows
$\sigma(x) = 0.1$
For the remaining vertices we have following cases.

Case i : if $n \equiv 0(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n}{3}$

$\sigma(x_k) = 0.3 \qquad \frac{n}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.1 \qquad 1 \leq k \leq \frac{n}{3}$

$\sigma(y_k) = 0.2 \qquad \frac{n}{3} + 1 \leq k \leq n$

Case ii : if $n \equiv 1(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n-1}{3}$

$\sigma(x_k) = 0.3 \qquad \frac{n-1}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.1 \qquad 1 \leq k \leq \frac{n-1}{3}$

$\sigma(y_k) = 0.2 \qquad \frac{n-1}{3} + 1 \leq k \leq n$

Case iii : if $n \equiv 2(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n+1}{3}$

$\sigma(x_k) = 0.3 \qquad \frac{n+1}{3} + 1 \leq k \leq n$

$\sigma(y_1) = 0.3$

$\sigma(y_k) = 0.1 \qquad 2 \leq k \leq \frac{n+1}{3}$

$\sigma(y_k) = 0.2 \qquad \frac{n+1}{3} + 1 \leq k \leq n$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
0	$\frac{2n}{3} + 1$	$\frac{2n}{3}$	$\frac{2n}{3}$
1	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
2	$\frac{2n+2}{3}$	$\frac{2n+2}{3} - 1$	$\frac{2n+2}{3}$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
1	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
2	$\frac{2n-1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-1}{3}$

From the above two tables it is concluded that the graph $S(G_{1,n})$ is fuzzy quotient-3 cordial. $\square$

Example 2.1. *The graph $S(G_{1,5})$ is fuzzy quotient-3 cordial.*

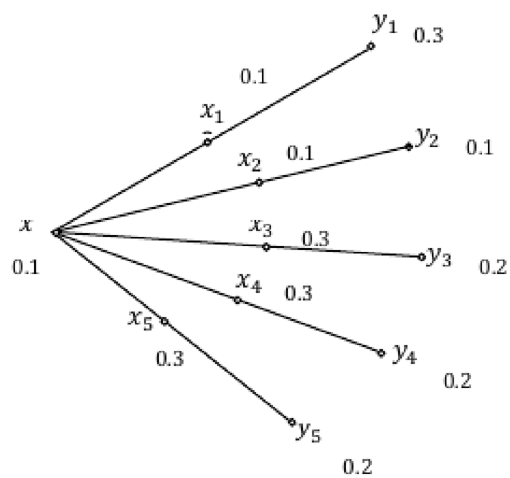

Figure 1.

Theorem 2.2. *The graph $S(G_{n,n})$ is fuzzy quotient-3 cordial.*

Proof. Let G be a $S(G_{n,n})$ graph.
Let $V(G) = \{u, w, v\} \cup \{u_k, v_k, x_k, y_k : 1 \leq k \leq n\}$ and
$E(G) = \{(u, w), (w, v)\} \cup \{(v, x_k) : 1 \leq k \leq n\} \cup \{(x_k, y_k) : 1 \leq k \leq n\} \cup \{(uu_k) : 1 \leq k \leq n\} \cup \{(u_k v_k) : 1 \leq k \leq n\}$.
Here $|V(G)| = 4n + 1$, $|E(G)| = 4n + 2$.
Define $\sigma : V(G) \to [0, 1]$ as follows by $\sigma(u) = \sigma(w) = \sigma(v) = 0.1$
$\sigma(u_k) = 0.3 \qquad 1 \leq k \leq n$.
$\sigma(v_k) = 0.2 \qquad 1 \leq k \leq n$.
For the remaining vertices we have following cases.

Case i : if $n \equiv 0(mod\ 3)$

$\sigma(x_k) = 0.3 \qquad 1 \leq k \leq \frac{n+3}{3}$

$\sigma(x_k) = 0.1 \qquad \frac{n+3}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n+3}{3}$

$\sigma(y_k) = 0.1 \qquad \frac{n+3}{3} + 1 \leq k \leq n$

Case ii : if $n \equiv 1(mod\ 3)$

$\sigma(x_k) = 0.3 \qquad 1 \leq k \leq \frac{n+2}{3}$

$\sigma(x_k) = 0.1 \qquad \frac{n+2}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n+2}{3}$

$\sigma(y_k) = 0.1 \qquad \frac{n+2}{3} + 1 \leq k \leq n$

Case iii : if $n \equiv 2(mod\ 3)$

$\sigma(x_k) = 0.3 \qquad 1 \leq k \leq \frac{n+1}{3}$

$\sigma(x_k) = 0.1 \qquad \frac{n+1}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n+4}{3}$

$\sigma(y_k) = 0.1 \qquad \frac{n+4}{3} + 1 \leq k \leq n$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
0	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$
1	$\frac{4n+2}{3} + 1$	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$
2	$\frac{4n+1}{3} + 1$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$\frac{4n}{3} + 1$	$\frac{4n}{3}$	$\frac{4n}{3} + 1$
1	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$	$\frac{4n+2}{3}$
2	$\frac{4n+1}{3}$	$\frac{4n+1}{3} + 1$	$\frac{4n+1}{3}$

From the above two tables it is concluded that the graph $S(G_{n,n})$ is fuzzy quotient-3 cordial. $\square$

Example 2.2. *The graph $S(G_{4,4})$ is Fuzzy quotient-3 cordial.*

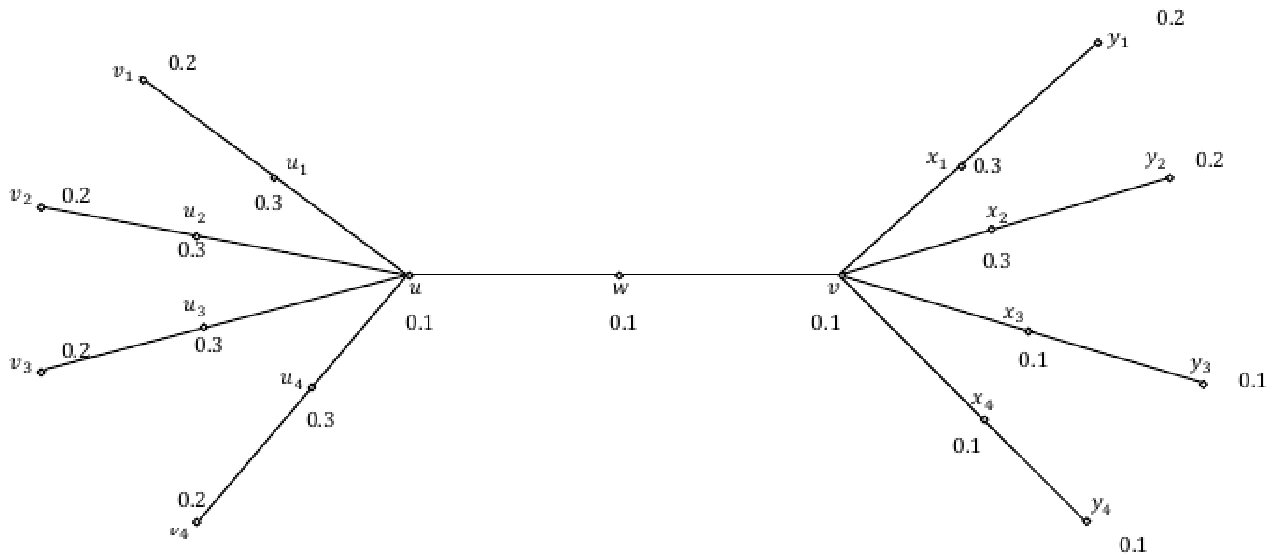

Figure 2.

Theorem 2.3. *The graph $S'(G_{1,n})$ is a Fuzzy quotient-3 cordial labeling.*

Proof. Let x the central vertex and x_k, $1 \leq k \leq n$ be the pendant vertices of the star graph $G_{1,n}$.
Let y and y_k $1 \leq k \leq n$ be the vertices corresponding to x and x_k, $1 \leq k \leq n$ to obtain $S'(G_{1,n})$. Let G be a $S'(G_{1,n})$ graph.
Let $V(G) = \{x\} \cup \{x_k : 1 \leq k \leq n\} \cup \{y\} \cup \{y_k : 1 \leq k \leq n\}$ and
$E(G) = \{(yx_k) : 1 \leq k \leq n\} \cup (yy_k) : 1 \leq k \leq n\} \cup \{(xy_k) : 1 \leq k \leq n\}$
Here $|V(G)| = 2n + 2$, $|E(G)| = 3n$.
Define $\sigma : V(G) \to [0,1]$ as follows
$\sigma(x) = 0.3$
$\sigma(y) = 0.1$
For the remaining vertices we have following cases.

Case i : if $n \equiv 0(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n}{3}$
$\sigma(x_k) = 0.2 \qquad \frac{n}{3} + 1 \leq k \leq \frac{2n}{3}$
$\sigma(x_k) = 0.3 \qquad \frac{2n}{3} + 1 \leq k \leq n$
$\sigma(y_k) = 0.1 \qquad 1 \leq k \leq \frac{n}{3}$
$\sigma(y_k) = 0.2 \qquad \frac{n}{3} + 1 \leq k \leq \frac{2n}{3}$
$\sigma(y_k) = 0.3 \qquad \frac{2n}{3} + 1 \leq k \leq n$

Case ii : if $n \equiv 1(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n-1}{3}$

$\sigma(x_k) = 0.2 \qquad \frac{n-1}{3} + 1 \leq k \leq \frac{2n+1}{3}$

$\sigma(x_k) = 0.3 \qquad \frac{2n+1}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.1 \qquad 1 \leq k \leq \frac{n+2}{3}$

$\sigma(y_k) = 0.2 \qquad \frac{n+2}{3} + 1 \leq k \leq \frac{2n+1}{3}$

$\sigma(y_k) = 0.3 \qquad \frac{2n+1}{3} + 1 \leq k \leq n$

Case iii : if $n \equiv 2(mod\ 3)$

$\sigma(x_k) = 0.1 \qquad 1 \leq k \leq \frac{n-2}{3}$

$\sigma(x_k) = 0.2 \qquad \frac{n-2}{3} + 1 \leq k \leq \frac{2n+2}{3}$

$\sigma(x_k) = 0.3 \qquad \frac{2n+2}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.1 \qquad 1 \leq k \leq \frac{n+1}{3}$

$\sigma(y_k) = 0.2 \qquad \frac{n+1}{3} + 1 \leq k \leq \frac{2n-1}{3}$

$\sigma(y_k) = 0.3 \qquad \frac{2n-1}{3} + 1 \leq k \leq n$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
0	$\frac{2n}{3} + 1$	$\frac{n}{3}$	$\frac{2n}{3} + 1$
1	$\frac{2n+1}{3} + 1$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
2	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	n	n	n
1	n	n	n
2	n	n	n

From the above two tables it is concluded that the graph $S'(G_{1,n})$ is Fuzzy quotient-3 cordial. $\square$

Example 2.3. *$S'(G_{1,4})$ is a fuzzy quotient-3 cordial labeling.*

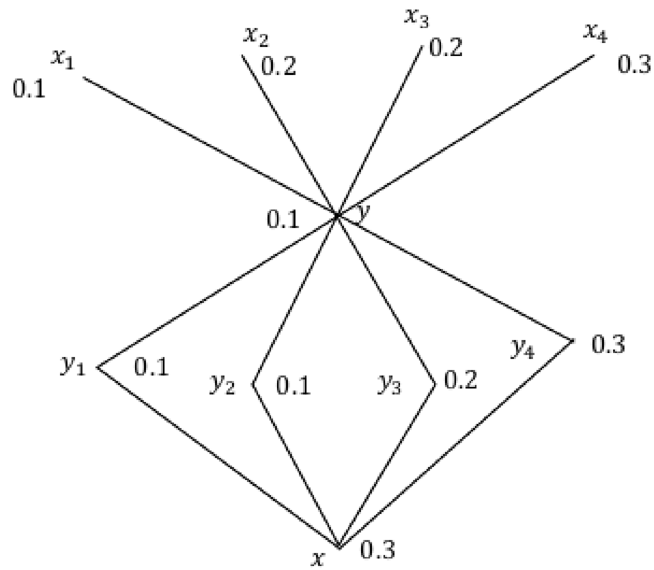

Figure 3.

Theorem 2.4. *The graph $S'(G_{n,n})$ is a fuzzy quotient-3 cordial labeling.*

Proof. Let x, y, x_k, y_k $1 \leq k \leq n$ be the vertices of the bistar graph $G_{n,n}$, where x_k's and y_k's are the pendant vertices adjacent to x and y respectively.
and u, v, u_k, v_k $1 \leq k \leq n$ be the vertices corresponding to x, y, x_k, y_k $1 \leq k \leq n$
Let G be a $S'(G_{n,n})$ graph.
Let $V(G) = \{x\} \cup \{x_k : 1 \leq k \leq n\} \cup \{y\} \cup \{y_k : 1 \leq k \leq n\} \cup \{v\} \cup \{v : 1 \leq k \leq n\}$ and
$E(G) = \{(xy)\} \cup \{(xx_k), 1 \leq k \leq n\} \cup \{(yy_k), 1 \leq k \leq n\} \cup \{(ux_k) : 1 \leq k \leq n\} \cup \{(vy_k) : 1 \leq k \leq n\} \cup \{(xu_k) : 1 \leq k \leq n\} \cup \{(yu_k) : 1 \leq k \leq n\} \cup \{(xv), (yu)\}$
Here $|V(G)| = 4(n+1)$, $|E(G)| = 6n+3$.
Define $\sigma : V(G) \to [0,1]$ as follows
$\sigma(x) = 0.1$
$\sigma(y) = 0.1$
$\sigma(u) = 0.3$
$\sigma(v) = 0.3$
For the remaining vertices we have following cases.

Case i : if $n \equiv 0(mod\ 3)$

$\sigma(x_k) = 0.2 \qquad 1 \leq k \leq \frac{n}{3}$
$\sigma(x_k) = 0.1 \qquad \frac{n}{3}+1 \leq k \leq n$
$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n}{3}$
$\sigma(y_k) = 0.3 \qquad \frac{n}{3}+1 \leq k \leq n$

$\sigma(u_k) = 0.2 \qquad 1 \leq k \leq \frac{n}{3} + 1$

$\sigma(u_k) = 0.1 \qquad \frac{n}{3} + 1 \leq k \leq n$

$\sigma(v_k) = 0.2 \qquad 1 \leq k \leq \frac{n}{3}$

$\sigma(v_k) = 0.1 \qquad i = \frac{n}{3} + 1$

$\sigma(v_k) = 0.3 \qquad \frac{n}{3} + 1 + 1 \leq k \leq n$

Case ii : if $n \equiv 1(mod\ 3)$

$\sigma(x_k) = 0.2 \qquad 1 \leq k \leq \frac{n+2}{3}$

$\sigma(x_k) = 0.1 \qquad \frac{n+2}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n-1}{3}$

$\sigma(y_k) = 0.3 \qquad \frac{n-1}{3} + 1 \leq k \leq n$

$\sigma(u_k) = 0.2 \qquad 1 \leq k \leq \frac{n+2}{3}$

$\sigma(u_k) = 0.1 \qquad \frac{n+2}{3} + 1 \leq k \leq n$

$\sigma(v_k) = 0.2 \qquad 1 \leq k \leq \frac{n-1}{3}$

$\sigma(v_k) = 0.1 \qquad k = \frac{n-1}{3} + 1$

$\sigma(v_k) = 0.3 \qquad \frac{n-1}{3} + 1 + 1 \leq k \leq n$

Case iii : if $n \equiv 2(mod\ 3)$

$\sigma(x_k) = 0.2 \qquad 1 \leq k \leq \frac{n+1}{3}$

$\sigma(x_k) = 0.1 \qquad \frac{n+1}{3} + 1 \leq k \leq n$

$\sigma(y_k) = 0.2 \qquad 1 \leq k \leq \frac{n-2}{3}$

$\sigma(y_k) = 0.3 \qquad \frac{n-2}{3} + 1 \leq k \leq n$

$\sigma(u_k) = 0.2 \qquad 1 \leq k \leq \frac{n+1}{3} + 1$

$\sigma(u_k) = 0.1 \qquad \frac{n+1}{3} + 1 + 1 \leq k \leq n$

$\sigma(v_k) = 0.2 \qquad 1 \leq k \leq \frac{n+1}{3}$

$\sigma(v_k) = 0.1 \qquad k = \frac{n+1}{3} + 1$

$\sigma(v_k) = 0.3 \qquad \frac{n+1}{3} + 11 \leq k \leq n$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$
0	$\frac{4(n+1)-1}{3} + 1$	$\frac{4(n+1)-1}{3}$	$\frac{4(n+1)-1}{3}$
1	$\frac{4(n+1)+1}{3}$	$\frac{4(n+1)+1}{3} - 1$	$\frac{4(n+1)+1}{3}$
2	$\frac{4(n+1)}{3}$	$\frac{4(n+1)}{3}$	$\frac{4(n+1)}{3}$

For $n \equiv l(mod\ 3)$, where $0 \leq l \leq 2$, the following table shows that the number of edges labeled with i and the number of edges labeled with j differ by at most 1, where $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

l	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
0	$2n+1$	$2n+1$	$2n+1$
1	$2n+1$	$2n+1$	$2n+1$
2	$2n+1$	$2n+1$	$2n+1$

From the above two tables it is concluded that the graph $S'(G_{n,n})$ is Fuzzy quotient-3 cordial. □

Example 2.4. *$S'(G_{4,4})$ is a fuzzy quotient-3 cordial labeling.*

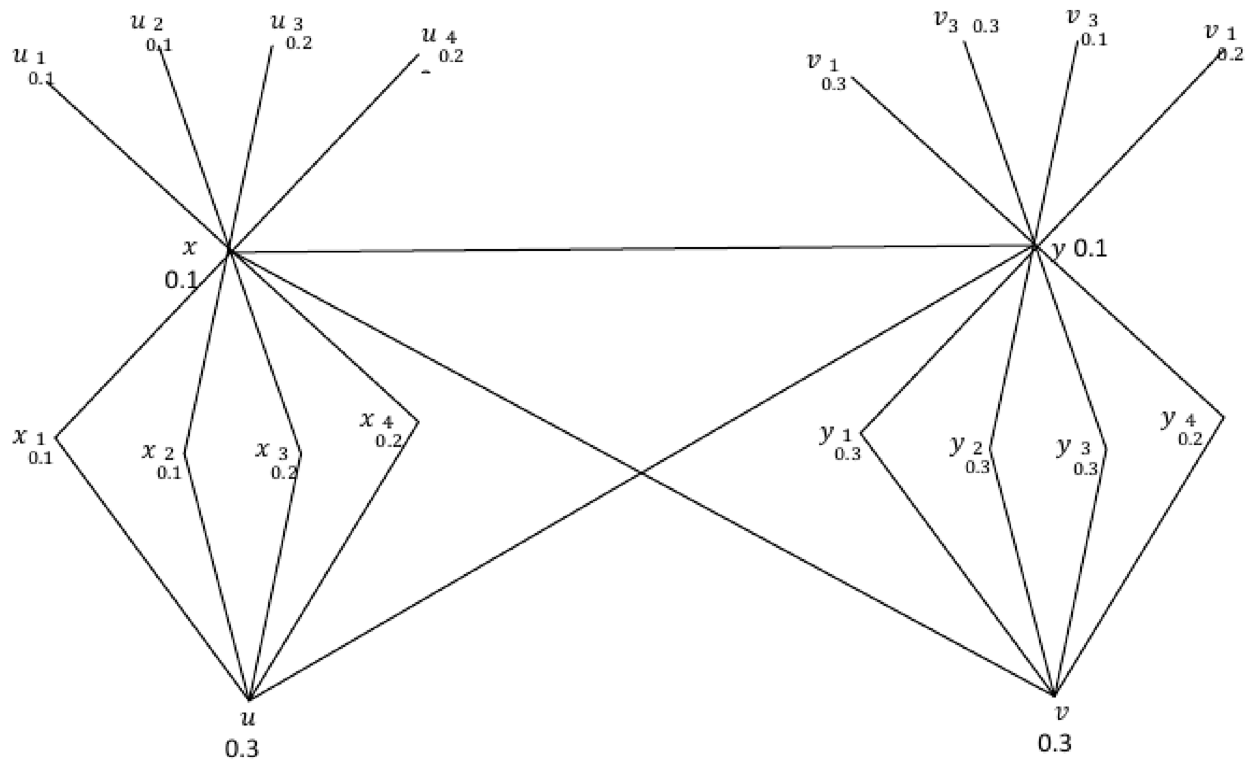

Figure 4.

3. Conclusion

In this paper, it is proved that Subdivision Star, Subdivision bistar, Splitting graph of star and bistar are Fuzzy quotient-3 cordial. The Fuzzy quotient-3 cordial labeling of different families of graphs shall be explored in future.

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