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An introductory notes to ideal binanotopological spaces

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Abstract:
In this paper, we made an attempt to unveil to notion of local functions, weak forms of $n_{1,2}$-open sets an ideal binanotopological spaces, $(1,2)^\star$ nano-I_g-closed sets, $(1,2)^\star$ nano $\star$-g-closed sets and $(1,2)^\star$ nano I_g-$\star$-closed sets are introduced and their properties are discussed with suitable examples. They are characterized in the context of an ideal binanotopological space.

Key words: local functions, weak forms of $n_{1,2}$-open sets and $(1,2)^\star$ nano-generalized closed sets

1. Introduction and Preliminaries

The notion of ideals in topological spaces has been studied by Kuratowski [6] and Vaidyanathaswamy [15, 16] which is one of the important areas of research in the branch of mathematics. Jankovic and Hamlett [4] further studied ideal topological spaces and their applications to various fields. Levine [8] introduced the notion of generalized closed sets in topological spaces. Donthev et al. [3] introduced and investigated the notion of I_g-closed sets in an ideal topological spaces as a modification of generalized closed sets. Navaneethakrishnan et al [9] investigated generalized closed sets in an ideal topological spaces and Kelly [5] introduced the concepts of bitopological spaces.

M. Lellis Thivagar et al [7] introduced the concept of nano topological spaces. I. Rajasekaran and O. Nethaji [13, 14] introduced $(1,2)^\star$ nano-b-open sets, $(1,2)^\star$ nano-β-open sets in binanotopological spaces and Simple forms of nano open sets in an ideal nanotopological spaces. K. Bhuvaneswari et al [1, 2] introduced the notions of $(1,2)^\star$ nano-α-open and $(1,2)^\star$ nano-pre-open sets in nano bitoplogical spaces and Nirmala Rebecca Paul [10] introduced on generalised closed sets in nano bitopological spaces.

In this paper, we made an attempt to unveil to notion of local functions, weak forms of $n_{1,2}$-open sets an ideal binanotopological spaces, $(1,2)^\star$ nano-I_g-closed sets, $(1,2)^\star$ nano $\star$-g-closed sets and $(1,2)^\star$ nano I_g-$\star$-closed sets are introduced and their properties are discussed with suitable examples. They are characterized in the context of an ideal binanotopological space.

In the rest of the paper, we denote a binanotopological space by $(U, \mathcal{N}_1, \mathcal{N}_2)$, where $\mathcal{N}_1 = \tau_R$ and

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$\mathcal{N}_2 = \tau_{R'}$. $\mathcal{N}_1 \cup \mathcal{N}_2 (or) \mathcal{N}_{1,2} = \tau_R \cup \tau_{R'}$. The binano-interior and binano-closure of a subset H of G are denoted by $n_{1,2}$-$int(H)$ and $n_{1,2}$-$cl(H)$, respectively.

A binanotopological space $(U, \mathcal{N}_1, \mathcal{N}_2)$ with an ideal I on U is called an ideal binanotopological space and is denoted by $(U, \mathcal{N}_1, \mathcal{N}_2, I)$.

Remark : ($N_{1,2}$-open or $n_{1,2}$-open) sets need not necessarily form a nanotopology.

2. $(\mathcal{N}_1, \mathcal{N}_2)$-local functions

In this section, we introduce the notion of $(\mathcal{N}_1, \mathcal{N}_2)$-local functions.

Given a binanotopological space $(U, \mathcal{N}_1, \mathcal{N}_2)$ with an ideal I on U, the $(\mathcal{N}_1, \mathcal{N}_2)$-local function of H with respect to $\mathcal{N}_1$, $\mathcal{N}_2$ and I by $H^\star_{n_{1,2}}(\mathcal{N}_1, \mathcal{N}_2, I) = \{u \in U | H \cap G \notin I$ for every $n_{1,2}$-open set containing u $\}$. $H^\star_{n_{1,2}}(\mathcal{N}_1, \mathcal{N}_2, I)$ are denoted by $H^\star_{n_{1,2}}$, respectively.

Theorem 2.1. *For subsets H, K of an ideal binanotopological space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$, the following properties are true:*

1. *If $H \subseteq K \Longrightarrow H^\star_{n_{1,2}} \subseteq K^\star_{n_{1,2}}$.*
2. *$H^\star_{n_{1,2}} = n_{1,2}$-$cl(H^\star) \subseteq n_{1,2}$-$cl(H)$.*
3. *$(H^\star_{n_{1,2}})^\star_{n_{1,2}} \subseteq H^\star_{n_{1,2}}$.*
4. *$(H \cup K)^\star_{n_{1,2}} = H^\star_{n_{1,2}} \cup K^\star_{n_{1,2}}$.*
5. *$V \in \mathcal{N}_{1,2} \Rightarrow V \cap H^\star_{n_{1,2}} = V \cap (V \cap H)^\star_{n_{1,2}} \subseteq (V \cap H)^\star_{n_{1,2}}$.*

Proof.

1. Assuming that $H \subseteq K$ and $u \notin K^\star_{n_{1,2}}$. Then, there exists a $n_{1,2}$-open set G containing u such that $G \cap K \in I$. Since $H \subseteq K$, we have $G \cap H \in I$ and so $u \notin H^\star_{n_{1,2}}$. This shows that $H^\star_{n_{1,2}} \subseteq K^\star_{n_{1,2}}$.
2. Let $u \in n_{1,2}$-$cl(H^\star)$. Then $H^\star_{n_{1,2}} \cap G \neq \phi$ for every $n_{1,2}$-open set G containing u. Therefore, there exists $v \in H^\star_{n_{1,2}} \cap G$. Since G containing v and $v \in H^\star_{n_{1,2}}$, we've $G \cap H \notin I$ and so $u \in H^\star_{n_{1,2}}$. Hence, $n_{1,2}$-$cl(H^\star) \subseteq H^\star_{n_{1,2}}$. As a result, we obtain $n_{1,2}$-$cl(H^\star) = H^\star_{n_{1,2}}$. Again, let $u \in n_{1,2}$-$cl(H^\star) = H^\star_{n_{1,2}}$, then $G \cap H \notin I$ for each $n_{1,2}$-open set G containing u. This implies that $G \cap H \neq \phi$ for every $n_{1,2}$-open set G containing u. Therefore, we have $u \in n_{1,2}$-$cl(H)$. This proves $H^\star_{n_{1,2}} = n_{1,2}$-$cl(A^\star) \subseteq n_{1,2}$-$cl(A)$.
3. Let $u \in (H^\star_{n_{1,2}})^\star_{n_{1,2}}$. Then for every $n_{1,2}$-open set G containing u, $G \cap H^\star_{n_{1,2}} \notin I$ and so $G \cap H^\star_{n_{1,2}} \neq \phi$. Therefore, there exists $v \in G \cap H^\star_{n_{1,2}}$. Since G containing v and $v \in H^\star_{n_{1,2}}$, we have $G \cap H \notin I$ and so $u \in H^\star_{n_{1,2}}$. This shows that $(H^\star_{n_{1,2}})^\star_{n_{1,2}} \subseteq H^\star_{n_{1,2}}$.
4. From (1), we have $H^\star_{n_{1,2}} \cup K^\star_{n_{1,2}} \subseteq (H \cup K)^\star_{n_{1,2}}$. For the reverse inclusion, suppose that $u \notin H^\star_{n_{1,2}} \cup K^\star_{n_{1,2}}$. Then, we have $u \notin H^\star_{n_{1,2}}$ and $u \notin K^\star_{n_{1,2}}$. There exist $n_{1,2}$-open set G containing u and $n_{1,2}$-open set A containing u such that $G \cap H \in I$ and $A \cap K \in I$.

$$\text{Therefore, } (G \cap A) \cap (H \cup K) = [(G \cap A) \cap H] \cup [(G \cap A) \cap K]$$
$$\subseteq (G \cap H) \cup (A \cap K) \in I$$
and so $u \notin (H \cup K)^\star_{n_{1,2}}$.

$\Rightarrow (H \cup K)^\star_{n_{1,2}} \subseteq H^\star_{n_{1,2}} \cup K^\star_{n_{1,2}}$. As a result, we find $(H \cup K)^\star_{n_{1,2}} = H^\star_{n_{1,2}} \cup K^\star_{n_{1,2}}$.

5. Assuming that $V \in \mathcal{N}_{1,2}$ and $u \in V \cap H^\star_{n_{1,2}}$. Then $u \in V$ and $u \in H^\star_{n_{1,2}}$. Since $V \in \mathcal{N}_{1,2}$, then there exists $K \in \mathcal{N}_{1,2}$ such $u \in K \subseteq n_{1,2}\text{-}cl(K) \subseteq V$. Let G be any $n_{1,2}$-open set containing u.
Followed by $G \cap K \in \mathcal{N}_{1,2}(u)$ and $n_{1,2}\text{-}cl(G \cap K) \cap H \notin I$ and hence $n_{1,2}\text{-}cl(G) \cap (V \cap H) \notin I$. This shows that $u \in (V \cap H)^\star_{n_{1,2}}$ and hence we find $V \cap H^\star_{n_{1,2}} \subseteq (V \cap H)^\star_{n_{1,2}}$. Also, $V \cap H^\star_{n_{1,2}} \subseteq V \cap (V \cap H)^\star_{n_{1,2}}$ and by (3) $(V \cap H)^\star_{n_{1,2}} \subseteq H^\star_{n_{1,2}}$ and $V \cap (V \cap H)^\star_{n_{1,2}} \subseteq V \cap H^\star_{n_{1,2}}$. Hence, $V \cap H^\star_{n_{1,2}} = V \cap (V \cap H)^\star_{n_{1,2}}$.

Definition 2.1. Let $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ be an ideal binanotopological space. For any subset H of U, we put $n_{1,2}\text{-}cl^\star(H) = H \cup H^\star_{n_{1,2}}$. The operator $n_{1,2}\text{-}cl^\star$ is a Kuratowski closure operator. The binano topology generated by $n_{1,2}\text{-}cl^\star$ is denoted by $\mathcal{N}^\star_{1,2}$ or $n^\star_{1,2}$, that is $\mathcal{N}^\star_{1,2} = \{G \subseteq U | n_{1,2}\text{-}cl^\star(U - G) = U - G\}$.

The elements of $\mathcal{N}^\star_{1,2}$ are called $n_{1,2}$-$\star$-open sets and the complement of a $n_{1,2}$-$\star$-open set is called $n_{1,2}$-$\star$-closed.

The closure and the interior of H with respect to $\mathcal{N}^\star_{1,2}$ or $n^\star_{1,2}$ are denoted by $n_{1,2}\text{-}cl^\star(H)$ and $n_{1,2}\text{-}int^\star(H)$, respectively.

Proposition 2.1. *For subsets H and K of an ideal binanotopological space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$, the following properties are true:*

1. $H \subseteq n_{1,2}\text{-}cl^\star(H)$.
2. $n_{1,2}\text{-}cl^\star(\phi) = \phi$ *and* $n_{1,2}\text{-}cl^\star(U) = U$.
3. *If* $H \subseteq K$, *then* $n_{1,2}\text{-}cl^\star(H) \subseteq n_{1,2}\text{-}cl^\star(K)$.
4. $n_{1,2}\text{-}cl^\star(H) \cup n_{1,2}\text{-}cl^\star(K) \subseteq n_{1,2}\text{-}cl^\star(H \cup K)$.

Definition 2.2. A subset H of an ideal binanotopological space is called

1. $n_{1,2}$-$\star$-closed if $H^\star_{n_{1,2}} \subseteq H$.
2. $n_{1,2}$-$\star$-dense in itself if $H \subseteq H^\star_{n_{1,2}}$.
3. $n_{1,2}$-$\star$-perfect if $H^\star_{n_{1,2}} = H$.

Proposition 2.2. *If H is $n_{1,2}$-$\star$-dense in itself in an ideal binanotopological space, then* $H^\star_{n_{1,2}} = n_{1,2}\text{-}cl(H^\star) = n_{1,2}\text{-}cl(H) = n_{1,2}\text{-}cl^\star(H)$.

Proof.

Assuming that H is $n_{1,2}$-$\star$-dense in itself. Then, we have $H \subseteq H^\star_{n_{1,2}}$ and so $n_{1,2}\text{-}cl(H) \subseteq n_{1,2}\text{-}cl(H^\star)$. By Proposition 2.1, $n_{1,2}\text{-}cl(H) \subseteq n_{1,2}\text{-}cl(H^\star) \subseteq n_{1,2}\text{-}cl(H)$ and hence $H^\star_{n_{1,2}} = n_{1,2}\text{-}cl(H^\star) = n_{1,2}\text{-}cl(H)$. Since $H^\star_{n_{1,2}} = n_{1,2}\text{-}cl(H)$, we have $n_{1,2}\text{-}cl^\star(H) = n_{1,2}\text{-}cl(H)$.

As a Result, we find $H^\star_{n_{1,2}} = n_{1,2}\text{-}cl(H^\star) = n_{1,2}\text{-}cl(H) = n_{1,2}\text{-}cl^\star(H)$.

Proposition 2.3. *For an ideal binanotopologial space, the following properties are true:*

1. *If H is $n_{1,2}$-open, then $n_{1,2}$-$int^\star(H) = H$.*
2. *If K is $n_{1,2}$-closed, then $n_{1,2}$-$cl^\star(K)$ is $n_{1,2}$-closed.*

Proof.

1. Let H be a $n_{1,2}$-open set. Then $U-H$ is $n_{1,2}$-closed, by Proposition 2.1, we have $(U-H)^\star_{n_{1,2}} \subseteq n_{1,2}$-$cl(U-H) = U-H$ and so $n_{1,2}$-$cl^\star(U-H) = U-H$. This implies that H is a $n_{1,2}$-$\star$-open set and hence $n_{1,2}$-$int^\star(H) = H$.
2. Let K be a $n_{1,2}$-closed set. Then, we have $n_{1,2}$-$cl(n_{1,2}$-$cl^\star(K)) = n_{1,2}$-$cl(K^\star \cup K) = n_{1,2}$-$cl(K^\star) \cup n_{1,2}$-$cl(K) = n_{1,2}$-$cl(K) = K \subseteq n_{1,2}$-$cl^\star(K)$. As a result, $n_{1,2}$-$cl(n_{1,2}$-$cl^\star(K)) = n_{1,2}$-$cl^\star(K)$ and so $n_{1,2}$-$cl^\star(K)$ is $n_{1,2}$-closed.

3. Weak forms of $n_{1,2}$-open sets: ideal binanotopological spaces

Definition 3.1. A subset H of an ideal binanotopological space is called

1. $(1,2)^\star$ nano-α-I-open if $H \subseteq n_{1,2}$-$int(n_{1,2}$-$cl^\star(n_{1,2}$-$int(H)))$,
2. $(1,2)^\star$ nano-semi-I-open if $H \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$,
3. $(1,2)^\star$ nano-pre-I-open if $H \subseteq n_{1,2}$-$int(n_{1,2}$-$cl^\star(H))$,
4. $(1,2)^\star$ nano-b-I-open if $H \subseteq n_{1,2}$-$int(n_{1,2}$-$cl^\star(H)) \cup n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$,
5. $(1,2)^\star$ nano-β-I-open if $H \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(n_{1,2}$-$cl^\star(H)))$.

The complements of the above used sets are following for their respective closed sets.

Theorem 3.1. *In a space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$, for a subset H, the following relations are true.*

1. *If H is $n_{1,2}$-open, then H is $(1,2)^\star$ nano-α-I-open.*
2. *If H is $(1,2)^\star$ nano-α-I-open, then H is nano $(1,2)^\star$-semi-I-open.*
3. *If H is $(1,2)^\star$ nano-α-I-open, then H is $(1,2)^\star$ nano-pre-I-open.*
4. *If H is $(1,2)^\star$ nano-semi-I-open, then H is $(1,2)^\star$ nano-b-I-open.*
5. *If H is $(1,2)^\star$ nano-pre-I-open, then H is $(1,2)^\star$ nano-b-I-open.*
6. *If H is $(1,2)^\star$ nano-b-I-open, then H is $(1,2)^\star$ nano-β-I-open.*

Proof.

1. H *is* $n_{1,2}$-open, then $H = n_{1,2}$-$int(H)$.

$$\text{But } H \subseteq n_{1,2}\text{-}cl^\star(H) = n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$$
$$\subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)))$$

which proves that H is $(1,2)^\star$ nano-α-I-open.

2. H *is* $(1,2)^\star$nano-α-I-open, then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)))$

$$\subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$$

which proves that H is $(1,2)^\star$ nano-semi-I-open.

3. H *is* $(1,2)^\star$nano-α-I-open, then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)))$

$$\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))$$

which proves that H is $(1,2)^\star$ nano-pre-I-open.

4. H *is* $(1,2)^\star$nano-*semi*-I-open, then $H \subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$

$$\subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)) \cup n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))$$

which proves that H is $(1,2)^\star$ nano-b-I-open.

5. H *is* $(1,2)^\star$nano-*pre*-I-open, then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))$

$$\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)) \cup n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$$

which proves that H is $(1,2)^\star$ nano-b-I-open.

6. H *is* $(1,2)^\star$nano-b-I-open, then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)) \cup n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$

$$\begin{aligned}&\subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)))\\ &\qquad \cup n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)))\\ &= n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)))\end{aligned}$$

which proves that H is $(1,2)^\star$ nano-β-I-open.

Remark 3.1. *These relations are shown in the diagram.*

$$\begin{array}{ccc} n_{1,2}\textbf{\textit{-open}} & & \\ \Downarrow & & \\ (1,2)^\star\textbf{\textit{nano-}}\alpha\textbf{\textit{-}}I\textbf{\textit{-open}} & \Longrightarrow & (1,2)^\star\textbf{\textit{nano-pre-}}I\textbf{\textit{-open}} \\ \Downarrow & & \Downarrow \\ (1,2)^\star\textbf{\textit{nano-semi-}}I\textbf{\textit{-open}} & \Longrightarrow & (1,2)^\star\textbf{\textit{nano-b-}}I\textbf{\textit{-open}} \\ & & \Downarrow \\ & & (1,2)^\star\textbf{\textit{nano-}}\beta\textbf{\textit{-}}I\textbf{\textit{-open}} \end{array}$$

Example 3.1. *Let* $U = \{m_1, m_2, m_3, m_4, m_5\}$ *with* $U/R = \{\{m_1\}, \{m_2, m_3\}, \{m_4, m_5\}\}$

1. $X_1 = \{m_1, m_2\}$, *then* $\mathcal{N}_1 = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$.

2. $X_2 = \{m_2, m_3\}$, *then* $\mathcal{N}_2 = \{\phi, U, \{m_2, m_3\}\}$.
   Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$.
   Let the ideal be $I = \{\phi, \{m_2\}\}$.

   (a) *the set* $\{m_1, m_2, m_3, m_4\}$ *which is* $(1,2)^\star$ *nano-*α*-*I*-open need not to be* $n_{1,2}$*-open.*

   (b) *the set* $\{m_2, m_3, m_4\}$ *is* $(1,2)^\star$ *nano-semi-*I*-open need not to be* $(1,2)^\star$ *nano* α*-*I*-open.*

   (c) *the set* $\{m_3, m_4\}$ *is* $(1,2)^\star$ *nano-*β*-*I*-open need not to be* $(1,2)^\star$ *nano-*b*-*I*-open.*

Example 3.2. *Let* $U = \{m_1, m_2, m_3, m_4\}$ *with* $U/R = \{\{m_1\}, \{m_3\}, \{m_2, m_4\}\}$

1. $X_1 = \{m_1, m_2\}$, *then* $\mathcal{N}_1 = \{\phi, U, \{m_1\}, \{m_2, m_4\}, \{m_1, m_2, m_4\}\}$.
2. $X_2 = \{m_1, m_2, m_4\}$, *then* $\mathcal{N}_2 = \{\phi, U, \{m_1, m_2, m_4\}\}$.
   Then binanotopologies $\mathcal{N}_{1,2} = \{\phi, U, \{m_1\}, \{m_2, m_4\}, \{m_1, m_2, m_4\}\}$.
   Let the ideal be $I = \{\phi, \{m_1\}\}$.

   (a) *the set* $\{m_2\}$ *is* $(1,2)^\star$ *nano pre-*I*-open need not to be* $(1,2)^\star$ *nano-*α*-*I*-open.*

   (b) *the set* $\{m_1, m_4\}$ *is* $(1,2)^\star$ *nano-*b*-*I*-open need not to be* $(1,2)^\star$ *nano-semi-*I*-open.*

   (c) *the set* $\{m_2, m_3, m_4\}$ *is* $(1,2)^\star$ *nano-*b*-*I*-open need not to be* $(1,2)^\star$ *nano-pre-*I*-open.*

Theorem 3.2. *A subset* H *of an ideal binanotopological space is* $(1,2)^\star$ *nano-*α*-*I*-open iff* H *is* $(1,2)^\star$ *nano-semi-*I*-open and* $(1,2)^\star$ *nano-pre-*I*-open.*

Proof.

Direct part follows from (2) and (3) of Theorem 3.1.

Indirect part of if H is $(1,2)^\star$ nano-semi-I-open and $(1,2)^\star$ nano-pre-I-open then

$H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))$ and $H \subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$.

Thus $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)) \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)))$

$$= n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)))$$

which proves that H is $(1,2)^\star$ nano-α-I-open.

Remark 3.2. *In a space* $(U, \mathcal{N}_1, \mathcal{N}_2, I)$, $(1,2)^\star$ *nano-semi-*I*-open sets and* $(1,2)^\star$ *nano-pre-*I*-open sets are independent of each other as the following Example show.*

Example 3.3. *Let* $U = \{m_1, m_2, m_3, m_4\}$ *with* $U/R = \{\{m_1\}, \{m_4\}, \{m_2, m_3\}\}$.

1. $X_1 = \{m_1, m_3\}$, *then* $\mathcal{N}_1 = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$ *and*
2. $X_2 = \{m_1\}$, *then* $\mathcal{N}_2 = \{\phi, U, \{m_1\}\}$.
   Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$.
   Let the ideal be $I = \{\phi, \{m_3\}\}$.

   (a) *the set* $\{m_1, m_3\}$ *is* $(1,2)^\star$ *nano-semi-*I*-open need not to be* $(1,2)^\star$ *nano-pre-*I*-open.*

   (b) *the set* $\{m_2\}$ *is* $(1,2)^\star$ *nano-pre-*I*-open need not to be* $(1,2)^\star$ *nano-semi-*I*-open.*

Theorem 3.3. *If a subset H of a space* $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ *is both* $n_{1,2}$*-*$\star$*-closed and* $(1,2)^\star$ *nano-*β*-*I*-open, then H is* $(1,2)^\star$ *nano-semi-*I*-open.*

Proof. Since H is $(1,2)^\star$ nano-β-I-open, $H \subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)))$

$$= n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)),$$

H being $n_{1,2}$-$\star$-closed. Therefore H is $(1,2)^\star$ nano-semi-I-open.

Theorem 3.4. *A subset H of a space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ is $(1,2)^\star$ nano-semi-I-open if and only if $n_{1,2}$-$cl^\star(H) = n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$.*

Proof. Let H be $(1,2)^\star$ nano-semi-I-open.

Then $H \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$ and $n_{1,2}$-$cl^\star(H) \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$.

But $n_{1,2}$-$cl^\star(n_{1,2}$-$int(H)) \subseteq n_{1,2}$-$cl^\star(H)$. Thus $n_{1,2}$-$cl^\star(H) = n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$.

Conversely, let the condition true. We have $H \subseteq n_{1,2}$-$cl^\star(H) = n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$, by assumption. Thus $H \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$ and hence H is $(1,2)^\star$ nano-semi-I-open.

Proposition 3.1. *In $(U, \mathcal{N}, I)$ if H is a $(1,2)^\star$ nano-b-I-open set such that $n_{1,2}$-$cl^\star(H) = \phi$, then H is $(1,2)^\star$ nano-semi-I-open.*

Theorem 3.5. *A subset H of an ideal binanotopological space is $(1,2)^\star$ nano-semi-I-open if and only if there exists a $n_{1,2}$-open set K such that $K \subseteq H \subseteq n_{1,2}$-$cl^\star(K)$.*

Proof.

Let H be $(1,2)^\star$ nano-semi-I-open. Then $H \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$. Take $n_{1,2}$-$int(H) = K$. Then $K \subseteq H \subseteq n_{1,2}$-$cl^\star(K)$, where K is $n_{1,2}$-open.

Conversely, let $K \subseteq H \subseteq n_{1,2}$-$cl^\star(K)$ for some $n_{1,2}$-open set K. Since $K \subseteq H$, $K \subseteq n_{1,2}$-$int(H)$ and $H \subseteq n_{1,2}$-$cl^\star(K) \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(H))$ which implies H is $(1,2)^\star$ nano-semi-I-open.

Theorem 3.6. *If H is a $(1,2)^\star$ nano-semi-I-open set in an ideal binanotopological space and $H \subseteq K \subseteq n_{1,2}$-$cl^\star(H)$, then K is $(1,2)^\star$ nano-semi-I-open.*

Proof.

By assumption $K \subseteq n_{1,2}$-$cl^\star(H) \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$cl^\star(n_{1,2}$-$int(H)))$ (for H is $(1,2)^\star$ nano-semi-I-open) $= n_{1,2}$-$cl^\star(n_{1,2}$-$int(H)) \subseteq n_{1,2}$-$cl^\star(n_{1,2}$-$int(K))$ by assumption. This implies K is $(1,2)^\star$ nano-semi-I-open.

Proposition 3.2. *For a subset of H an ideal binanotopological space, the following properties are true:*

1. *If H is α-I-open, then H is $(1,2)^\star$ nano-α-open.*
2. *If H is $(1,2)^\star$ nano-pre-I-open, then H is $(1,2)^\star$ nano-pre-open.*
3. *If H is $(1,2)^\star$ nano-b-I-open, then H is $(1,2)^\star$ nano-b-open.*
4. *If H is $(1,2)^\star$ nano-β-I-open, then H is $(1,2)^\star$ nano-β-open.*

Proof.

1. Let H be a $(1,2)^\star$ nano-α-I-open set. Then
$$H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H)))$$
$$\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl(n_{1,2}\text{-}int(H))).$$
This shows that H is $(1,2)^\star$ nano-α-open.

2. Let H be a $(1,2)^\star$ nano-pre-I-open set. Then
$$H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))$$
$$\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl(H)).$$
This shows that H is $(1,2)^\star$ nano-pre-open.

3. Let H be a $(1,2)^\star$ nano-b-I-open set. Then
$$H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H)) \cup n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(H))$$
$$\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl(H)) \cup n_{1,2}\text{-}cl(n_{1,2}\text{-}int(H)).$$
This shows that H is $(1,2)^\star$ nano-b-open.

4. Let H be a $(1,2)^\star$ nano-β-I-open set.
Then $H \subseteq n_{1,2}\text{-}cl^\star(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^\star(H))) \subseteq n_{1,2}\text{-}cl(n_{1,2}\text{-}int(n_{1,2}\text{-}cl(H)))$.
This shows that H is $(1,2)^\star$ nano-β-open.

Remark 3.3. *The converses of Proposition 3.2 are not true in general as shown in the following Example.*

Example 3.4. *Let $U = \{m_1, m_2, m_3, m_4, m_5\}$ with $U/R = \{\{m_1\}, \{m_2, m_3\}, \{m_4, m_5\}\}$.*

1. *$X_1 = \{m_1, m_2\}$, then $\mathcal{N}_1 = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$.*

2. *$X_2 = \{m_1\}$, then $\mathcal{N}_2 = \{\phi, U, \{m_1\}\}$.*
Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, U, \{m_1\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}\}$.
Let the ideal be $I = \wp(U)$.
The set $\{m_1, m_2, m_3, m_4\}$ is $(1,2)^\star$ nano-α-open need not to be $(1,2)^\star$ nano-α-I-open.

Example 3.5. *Let $U = \{m_1, m_2, m_3\}$ with $U/R = \{\{m_1\}, \{m_2, m_3\}\}$*

1. *$X_1 = \{m_1, m_2\}$, then $\mathcal{N}_1 = \{\phi, U, \{m_1\}, \{m_2, m_3\}\}$.*

2. *$X_2 = \{m_2, m_3\}$, then $\mathcal{N}_2 = \{\phi, U, \{m_2, m_3\}\}$.*
Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, U, \{m_1\}, \{m_2, m_3\}\}$.
Let the ideal be $I = \{\phi, m_2\}$.
The set $\{m_1, m_2\}$ is $(1,2)^\star$ nano-pre-open need not to be $(1,2)^\star$ nano-pre-I-open.

Example 3.6. *In Example 3.2,*

1. *the set $\{m_1, m_3\}$ is $(1,2)^\star$ nano-b-open need not to be $(1,2)^\star$ nano-b-I-open.*

2. *the set $\{m_1, m_3\}$ is $(1,2)^\star$ nano-β-open need not to be $(1,2)^\star$ nano-β-I-open.*

Lemma 3.1. *Let $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ be a space and H a subset of U. If H is $n_{1,2}$-open in an ideal binanotopological space, then $K \cap n_{1,2}\text{-}cl^\star(H) \subseteq n_{1,2}\text{-}cl^\star(K \cap H)$.*

Proof.

$$
\begin{aligned}
K \cap\ n_{1,2}\text{-}cl^{\star}(H) &= K \cap (H^{\star}_{n_{1,2}} \cup H) \\
&= (K \cap H^{\star}_{n_{1,2}}) \cup (K \cap H) \\
&\subseteq (K \cap H)^{\star}_{n_{1,2}} \cup (K \cap H) \text{ by (5) of Theorem 2.1.} \\
\text{Thus } K \cap n_{1,2}\text{-}cl^{\star}(H) &\subseteq (K \cap H)^{\star}_{n_{1,2}} \cup (K \cap H) \\
&= n_{1,2}\text{-}cl^{\star}(K \cap H).
\end{aligned}
$$

Proposition 3.3. *The intersection of a $(1,2)^{\star}$ nano-pre-I-open set and $n_{1,2}$-open set is $(1,2)^{\star}$ nano-pre-I-open.*

Proof.

Let H be $(1,2)^{\star}$ nano-pre-I-open and G be $n_{1,2}$-open.
Then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))$ and

$$
\begin{aligned}
G \cap H &\subseteq n_{1,2}\text{-}int(G) \cap n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H)) \\
&= n_{1,2}\text{-}int(G \cap n_{1,2}\text{-}cl^{\star}(H)) \\
&\subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(G \cap H))
\end{aligned}
$$

by Lemma 3.1. This shows that $G \cap H$ is $(1,2)^{\star}$ nano-pre-I-open.

Proposition 3.4. *The intersection of a $(1,2)^{\star}$ nano-semi-I-open set and $n_{1,2}$-open set is $(1,2)^{\star}$ nano-semi-I-open.*

Proof.

Let H be $(1,2)^{\star}$ nano-semi-I-open and G be $n_{1,2}$-open in U.
Then $H \subseteq n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H))$ and $n_{1,2}\text{-}int(G) = G$.

$$
\begin{aligned}
G \cap H &\subseteq G \cap n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H)) \\
&\subseteq n_{1,2}\text{-}cl^{\star}(G \cap n_{1,2}\text{-}int(H)) \\
&= n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(G) \cap n_{1,2}\text{-}int(H)) \\
&= n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(G \cap H))
\end{aligned}
$$

by Lemma 3.1. Hence H is $(1,2)^{\star}$ nano-semi-I-open.

Proposition 3.5. *The intersection of a $(1,2)^{\star}$ nano-α-I-open set and $n_{1,2}$-open set is $(1,2)^{\star}$ nano-α-I-open.*

Proof.

Let G be a $n_{1,2}$-open and H be an $(1,2)^{\star}$ nano-α-I-open in a space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$. Then H is both $(1,2)^{\star}$ nano-pre-I-open and $(1,2)^{\star}$ nano-semi-I-open by (2) and (3) of Theorem 3.1. $H \cap G$ is both $(1,2)^{\star}$ nano-pre-I-open and $(1,2)^{\star}$ nano-semi-I-open by Proposition 3.3 and 3.4. Hence by Theorem 3.2, $H \cap G$ is $(1,2)^{\star}$ nano-α-I-open.

Proposition 3.6. *The intersection of a $(1,2)^{\star}$ nano-b-I-open set and $n_{1,2}$-open set is $(1,2)^{\star}$ nano-b-I-open.*

Proof.

Let H be $(1,2)^{\star}$ nano-b-I-open and G be $n_{1,2}$-open.
Then $H \subseteq n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H)) \cup n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H))$

$$\begin{aligned} \text{and } G \cap H &\subseteq G \cap [n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H)) \cup n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H))] \\ &= [G \cap n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))] \cup [G \cap n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H))] \\ &= [n_{1,2}\text{-}int(G) \cap n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))] \cup [G \cap n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(H))] \\ &\subseteq [n_{1,2}\text{-}int(G \cap n_{1,2}\text{-}cl^{\star}(H))] \cup [n_{1,2}\text{-}cl^{\star}(G \cap n_{1,2}\text{-}int(H))] \end{aligned}$$

by Lemma 3.1.

Thus $G \cap H \subseteq [n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(G \cap H))] \cup [n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(G \cap H))]$.

This shows that $G \cap H$ is $(1,2)^{\star}$ nano-b-I-open.

Proposition 3.7. *The intersection of a $(1,2)^{\star}$ nano-β-I-open set and $n_{1,2}$-open set is $(1,2)^{\star}$ nano-β-I-open.*

Proof.

Let H be $(1,2)^{\star}$ nano-β -I -open and G be $n_{1,2}$-open. Then

$$H \subseteq n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H)))$$

$$\begin{aligned} \text{and } G \cap H &\subseteq G \cap n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))) \\ &\subseteq n_{1,2}\text{-}cl^{\star}(G \cap n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))) \\ &\subseteq n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(G) \cap n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(H))) \\ &= n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(G \cap n_{1,2}\text{-}cl^{\star}(H))) \\ &\subseteq n_{1,2}\text{-}cl^{\star}(n_{1,2}\text{-}int(n_{1,2}\text{-}cl^{\star}(G \cap H))) \end{aligned}$$

by Lemma 3.1. This shows that $G \cap H$ is $(1,2)^{\star}$ nano-β-I-open.

Remark 3.4. *The intersection of two $(1,2)^{\star}$ nano-semi-I-open (resp. $(1,2)^{\star}$ nano-pre-I-open, $(1,2)^{\star}$ nano-b-I-open, $(1,2)^{\star}$ nano-β-I-open) sets need not be $(1,2)^{\star}$ nano-semi-I-open (resp. $(1,2)^{\star}$ nano-pre-I-open, $(1,2)^{\star}$ nano-b-I-open, $(1,2)^{\star}$ nano-β-I-open) as shown in the following Example.*

Example 3.7. *In Example 3.3, $\{m_1, m_4\}$ and $\{m_2, m_3, m_4\}$ the both are sets $(1,2)^{\star}$ nano-semi-I-open. Hence $\{m_4\}$ is not $(1,2)^{\star}$ nano-semi-I-open.*

Example 3.8. *In Example 3.2,*

1. *$\{m_1, m_2, m_3\}$ and $\{m_1, m_3, m_4\}$ the both are sets $(1,2)^{\star}$ nano-pre-nI-open and $(1,2)^{\star}$ nano-b-I-open. Hence $\{m_1, m_3\}$ is not $(1,2)^{\star}$ nano-pre-I-open and $(1,2)^{\star}$ nano-b-nI-open.*
2. *$\{m_2, m_3\}$ and $\{m_3, m_4\}$ the both are sets $(1,2)^{\star}$ nano-β-I-open. Hence $\{m_3\}$ is not $(1,2)^{\star}$ nano-β-I-open.*

4. $(1,2)^{\star}$ nano generalized closed sets : ideal binanotopological spaces

Definition 4.1. A subset H of a space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ is said to be

1. $(1,2)^{\star}$ nano-I_g-closed if $H^{\star}_{n_{1,2}} \subseteq K$ whenever $H \subseteq K$ and K is $n_{1,2}$-open.
2. $(1,2)^{\star}$ nano $\star$-g-closed if $n_{1,2}\text{-}cl(H) \subseteq K$ whenever $H \subseteq K$ and K is $n_{1,2}$-$\star$-open.

3. $(1,2)^{\star}$ nano I_g-$\star$-closed if $H^{\star}_{n_{1,2}} \subseteq K$ whenever $H \subseteq K$ and K is $n_{1,2}$-$\star$-open.

The complement of the above used sets are following for their respective open sets.

Theorem 4.1. *If H is a $n_{1,2}$-$\star$-closed subset of G in an ideal binanotopological space and K is $n_{1,2}$-closed in binanotopological space, then $H \cap K$ is $n_{1,2}$-$\star$-closed in an ideal binanotopological spaces.*

Proof.

$n_{1,2}$-$cl^{\star}(H \cap K) \subseteq n_{1,2}$-$cl^{\star}(H) \cap n_{1,2}$-$cl^{\star}(K) \subseteq n_{1,2}$-$cl^{\star}(H) \cap n_{1,2}$-$cl(K) = H \cap K$. Hence $H \cap K = n_{1,2}$-$cl^{\star}(H \cap K)$ and thus $H \cap K$ is $n_{1,2}$-$\star$-closed.

Theorem 4.2. *If any ideal binanotopological space and $H \subseteq G$, then the following are equivalent.*

1. *H is $(1,2)^{\star}$ nano I_g-$\star$-closed.*
2. *$n_{1,2}$-$cl^{\star}(H) \subseteq K$ whenever $H \subseteq K$ and K is $n_{1,2}$-$\star$-open.*
3. *$n_{1,2}$-$cl^{\star}(H) - H$ contains no nonempty $n_{1,2}$-$\star$-closed set.*
4. *$H^{\star}_{n_{1,2}} - H$ contains no nonempty $n_{1,2}$-$\star$-closed set.*

Proof.

(1) $\Rightarrow$ (2) Let $H \subseteq K$ where K is $n_{1,2}$-$\star$-open in G. Since H is $(1,2)^{\star}$ nano I_g-$\star$-closed, $H^{\star}_{n_{1,2}} \subseteq K$ and so $n_{1,2}$-$cl^{\star}(H) = H \cup H^{\star}_{n_{1,2}} \subseteq K$.

(2) $\Rightarrow$ (3) Let S be a $n_{1,2}$-$\star$-closed subset such that $S \subseteq n_{1,2}$-$cl^{\star}(H) - H$. Then $S \subseteq n_{1,2}$-$cl^{\star}(H)$. Moreover $S \subseteq n_{1,2}$-$cl^{\star}(H) - H \subseteq G - H$ and hence $H \subseteq G - S$ where $G - S$ is $n_{1,2}$-$\star$-open. By (2) $n_{1,2}$-$cl^{\star}(H) \subseteq G - S$ and so $S \subseteq G - n_{1,2}$-$cl^{\star}(H)$. Thus $S \subseteq n_{1,2}$-$cl^{\star}(H) \cap G - n_{1,2}$-$cl^{\star}(H) = \phi$. Hence $S = \phi$ and (3) is proved.

(3) $\Rightarrow$ (4) $H^{\star}_{n_{1,2}} - H = H \cup H^{\star}_{n_{1,2}} - H = n_{1,2}$-$cl^{\star}(H) - H$ which contains no nonempty $n_{1,2}$-$\star$-closed subset by (3).

(4) $\Rightarrow$ (1) Let $H \subseteq K$ where K is $n_{1,2}$-$\star$-open. Then $G - K \subseteq G - H$ and so $H^{\star}_{n_{1,2}} \cap (G - K) \subseteq H^{\star}_{n_{1,2}} \cap (G - H) = H^{\star}_{n_{1,2}} - H$. Since $H^{\star}_{n_{1,2}}$ is always $n_{1,2}$-closed and $G - K$ is $n_{1,2}$-$\star$-closed, $H^{\star}_{n_{1,2}} \cap$ (G − K) is $n_{1,2}$-$\star$-closed by Theorem 4.1 and also contained in $H^{\star}_{n_{1,2}} - H$. Hence $H^{\star}_{n_{1,2}} \cap (G - K) = \phi$. by (4). Thus $H^{\star}_{n_{1,2}} \subseteq K$ and H is $(1,2)^{\star}$ nano I_g-$\star$-closed.

Theorem 4.3. *In an ideal binanotopological space,*

1. *H is $n_{1,2}$-$\star$-closed, then H is $(1,2)^{\star}$ nano I_g-$\star$-closed.*
2. *$H \in I$, then H is $(1,2)^{\star}$ nano I_g-$\star$-closed.*
3. *H is $(1,2)^{\star}$ nano I_g-$\star$-closed, $n_{1,2}$-$\star$-open, then H is $n_{1,2}$-$\star$-closed.*
4. *H is $n_{1,2}$-$\star$-g-closed, then H is $(1,2)^{\star}$ nano I_g-$\star$-closed.*

Proof.

1. Let H be $n_{1,2}$-$\star$-closed set. To prove H is $(1,2)^{\star}$ nano I_g-$\star$-closed, let K be any $n_{1,2}$-$\star$-open set such that $H \subseteq K$. Since H is $n_{1,2}$-$\star$-closed, $H^{\star}_{n_{1,2}} \subseteq H \subseteq K$. Thus H is $(1,2)^{\star}$ nano I_g-$\star$-closed.

2. Let $H \in I$ and let $H \subseteq K$ where K is $n_{1,2}$-$\star$-open. Since $H \in I$, $H^\star_{n_{1,2}} = \phi \subseteq K$. Thus H is $(1,2)^\star$ nano I_g-$\star$-closed.

3. Let H be $(1,2)^\star$ nano I_g-$\star$-closed and $n_{1,2}$-$\star$-open. We have $H \subseteq H$ where H is $n_{1,2}$-$\star$-open. Since H is $(1,2)^\star$ nano I_g-$\star$-closed, $H^\star_{n_{1,2}} \subseteq H$. Thus H is $n_{1,2}$-$\star$-closed.

4. Let H be $n_{1,2}$-$\star$-g-closed and K any $n_{1,2}$-$\star$-open set such that $H \subseteq K$. Since H is $n_{1,2}$-$\star$-g-closed, $n_{1,2}$-$cl(H) \subseteq K$. So, $H^\star_{n_{1,2}} \subseteq n_{1,2}$-$cl(H) \subseteq K$ and thus H is $(1,2)^\star$ nano I_g-$\star$-closed.

Remark 4.1. *We have the following implications for a subset in the an ideal binanotopological space $(U, \mathcal{N}_1, \mathcal{N}_2, I)$ from the results stated above.*

$$\begin{array}{ccccc}
n_{1,2}\text{-}\textbf{\textit{closed}} & \Longrightarrow & (1,2)^\star \textbf{\textit{nano}}\ \star\text{-}g\text{-}\textbf{\textit{closed}} & \Longrightarrow & (1,2)^\star \textbf{\textit{nano}}\ g\text{-}\textbf{\textit{closed}} \\
\Downarrow & & \Downarrow & & \Downarrow \\
n_{1,2}\text{-}\star\text{-}\textbf{\textit{closed}} & \Longrightarrow & (1,2)^\star \textbf{\textit{nano}}\ I_g\text{-}\star\text{-}\textbf{\textit{closed}} & \Longrightarrow & (1,2)^\star \textbf{\textit{nano}}\ I_g\text{-}\textbf{\textit{closed}}
\end{array}$$

Example 4.1. *Let $U = \{m_1, m_2, m_3, m_4\}$ with $U/R = \{\{m_2\}, \{m_4\}, \{m_1, m_3\}\}$.*

1. $X_1 = \{m_3, m_4\}$, then $\mathcal{N}_1 = \{\phi, \{m_4\}, \{m_1, m_3\}, \{m_1, m_3, m_4\}, U\}$ and

2. $X_2 = \{m_4\}$, then $\mathcal{N}_2 = \{\phi, \{m_4\}, U\}$.
Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, \{m_4\}, \{m_1, m_3\}, \{m_1, m_3, m_4\}, U\}$.
Let the ideal be $I = \{\phi, \{d\}\}$.

(a) the set $\{m_4\}$ is $n\star$-closed, nI_g-closed and nI_g-$\star$-closed but not n-closed, ng-closed and $n\star$-g-closed.

(b) the set $\{m_1, m_2, m_4\}$ is nI_g-$\star$-closed and $n\star$-g-closed but not $n\star$-closed and n-closed.

(c) for $I = \wp(X)$, the set $\{m_1, m_2\}$ is ng-closed but not $n\star$-g-closed.

Example 4.2. *Let $U = \{m_1, m_2, m_3, m_4\}$ with $U/R = \{\{m_1, m_3\}, \{m_2, m_4\}\}$*

1. $X_1 = \{m_1, m_3, m_4\}$, then $\mathcal{N}_1 = \{\phi, \{m_2, m_4\}, \{m_1, m_3\}, U\}$ and

2. $X_2 = \{m_1, m_3\}$, then $\mathcal{N}_2 = \{\phi, \{m_1, m_3\}, U\}$.
Then the binanotopologies $\mathcal{N}_{1,2} = \{\phi, \{m_2, m_4\}, \{m_1, m_3\}, U\}$.
Let the ideal be $I = \{\phi, \{m_4\}\}$.
Hence, the set $\{m_2\}$ is nI_g-closed but not nI_g-$\star$-closed.

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