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Unified approach of several sets in ideal nanotopological spaces

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Abstract:
In this paper, further study is carried out using the established generalized classes of $\mathcal{N}$ in an ideal nanotopological spaces. Apart from this, several new generalizations of nano open sets be introduced and investigated.

Key words: semi-nI-regular set, pre-nI-regular set and β-nI-regular set.

1. Introduction and Preliminaries

During this paper, R. Vaidyanathaswamy (1945, 1946) and K. Kuratowski (1966) Independly introduced the concept an ideals. Z. Pawlak (1982) introduced lower approximation, upper approximation and boundary are investigated and M. Lellis Thivagar et al. (2013) introduced the notions of weaker forms of nanotopological spaces.

Some new notions in the concept of ideal nanotopological spaces were introduced by Parimala et al. [4, 5].

Recently, Rajasekaran et. al (2018) introduced the notions of weaker form of nano α-I-open sets, nano β-I-open sets and nano semi-I-open sets and nano t-I-set, nano t_α-I-set and nano $\mathcal{R}$-I-sets were studied in detail.

In present paper, further study is carried out using the established generalized classes of $\mathcal{N}$ in an ideal nanotopological spaces. A part from this, several new generalizations of nano open sets are introduced and investigated.

2. On new subsets of an ideal nano sets

Definition 2.1. A subset A of an ideal nanotopological space $(U, \mathcal{N}, I)$ is called a

1. nano $t^\#$-I-set (briefly, $t^\#$-nI-set) if $n\text{-}int(A) = n\text{-}cl^\star(n\text{-}int(A))$.
2. nano $t^\#_\alpha$-I-set (briefly, $t^\#_\alpha$-nI-set) if $n\text{-}int(A) = n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$.
3. nano $\mathcal{R}^\#$-I-set (briefly, $\mathcal{R}^\#$-nI-set) if $A = P \cap Q$, where P is n-open & Q is $t^\#$-nI-set.
4. nano $\mathcal{R}^\#_\alpha$-I-set (briefly, $\mathcal{R}^\#_\alpha$-nI-set) if $A = P \cap Q$, where P is n-open & Q is $t^\#_\alpha$-nI-set.

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5. strong nano $\mathcal{R}$-I-set (briefly, strong $\mathcal{R}$-nI-set) if $A = P \cap Q$, where P is n-open & Q is t-nI-set and $n\text{-}int(n\text{-}cl^\star(Q)) = n\text{-}cl^\star(n\text{-}int(Q))$.

Example 2.1. *Let $U = \{m_1, m_2, m_3, m_4\}$ with $U/R = \{\{m_2\}, \{m_4\}, \{m_1, m_3\}\}$ and $X = \{m_3, m_4\}$. Then $\mathcal{N} = \{\phi, \{m_4\}, \{m_1, m_3\}, \{m_1, m_3, m_4\}, U\}$ and $I = \{\phi, \{m_4\}\}$. Then $\{m_2\}$ is $t^\#$-nI-set, $t^\#_\alpha$-nI-set, $\mathcal{R}^\#$-nI-set, $\mathcal{R}^\#_\alpha$-nI-set and strong $\mathcal{R}$-nI-set.*

Remark 2.1. *An ideal nanotopological space $(U, \mathcal{N}, I)$,*

1. *each n-open set is $\mathcal{R}^\#$-nI-set.*
2. *each $t^\#$-nI-set is $\mathcal{R}^\#$-nI-set.*

Remark 2.2. *The converse in every part of the Remark 2.1 is need not be true as shown in the next two Examples.*

Example 2.2. *In Example 2.1,*

1. *$A = \{m_1\}$ is $\mathcal{R}^\#$-nI-set but not n-open.*
2. *$B = \{m_1, m_3, m_4\}$ is $\mathcal{R}^\#$-nI-set but not $t^\#$-nI-set.*

Proposition 2.1. *If P and Q are $t^\#$-nI-sets of an ideal nanotopological space $(U, \mathcal{N}, I)$, then $P \cap Q$ is $t^\#$-nI-set.*

Proof.

Let P and Q be $t^\#$-nI-sets. $n\text{-}int(P \cap Q) \subseteq n\text{-}int(P \cap Q) \subseteq n\text{-}cl^\star(n\text{-}int(P \cap Q)) = n\text{-}cl^\star(n\text{-}int(P) \cap n\text{-}int(Q)) \subseteq n\text{-}cl^\star(n\text{-}int(P)) \cap n\text{-}cl^\star(n\text{-}int(Q)) = n\text{-}int(P) \cap n\text{-}int(Q)$ (by guess) $= n\text{-}int(P \cap Q)$.
Thus $n\text{-}int(P \cap Q) = n\text{-}cl^\star(n\text{-}int(P \cap Q))$ and hence $P \cap Q$ is $t^\#$-nI-set.

Theorem 2.1. *For a subset A of an ideal nanotopological space $(U, \mathcal{N}, I)$, the following properties are equivalent:*

1. *A is n-open,*
2. *A is semi-nI-open and $\mathcal{R}^\#$-nI-set.*

Proof.

(1)$\Rightarrow$(2): (2) follows by Remark 3.3 of [7] and (1) of Remark 2.1.

(2)$\Rightarrow$(1): Given A is $\mathcal{R}^\#$-nI-set. So $A = P \cap Q$ where P is n-open and $n\text{-}int(Q) = n\text{-}cl^\star(n\text{-}int(Q))$. Then $A \subseteq P = n\text{-}int(P)$. Also A is semi-nI-open implies $A \subseteq n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(n\text{-}int(Q)) = n\text{-}int(Q)$ by guess. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(P)$ and hence A is n-open.

Remark 2.3. *An ideal nanotopological space the notions of $\mathcal{R}^\#$-nI-sets and of semi-nI-open sets are independent.*

Example 2.3. *In Example 2.1,*

1. *$A = \{m_1, m_2, m_3\}$ is semi-nI-open set but not $\mathcal{R}^\#$-nI-set.*
2. *$B = \{m_2\}$ is $\mathcal{R}^\#$-nI-set but not semi-nI-open.*

Remark 2.4. *An ideal nanotopological space* $(U, \mathcal{N}, I)$,

1. *each* n*-open set is* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set.*
2. *each* $t^{\#}_{\alpha}$*-*nI*-set is* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set.*

Remark 2.5. *The converse in every part of the Remark 2.4 is need not be true as shown in the next two Examples.*

Example 2.4. *In Example 2.1,*

1. $A = \{m_2\}$ *is* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set but not* n*-open.*
2. $B = \{m_1, m_3\}$ *is* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set but not* $t^{\#}_{\alpha}$*-*nI*-set.*

Proposition 2.2. *If* P *and* Q *are* $t^{\#}_{\alpha}$*-*nI*-sets of a space* $(U, \mathcal{N}, I)$, *then* $P \cap Q$ *is* $t^{\#}_{\alpha}$*-*nI*-set.*

Proof.

Let P and Q be $t^{\#}_{\alpha}$-nI-sets. $n\text{-}int(P \cap Q) \subseteq n\text{-}int(P \cap Q) \subseteq n\text{-}int(n\text{-}cl^{\star}(P \cap Q)) \subseteq n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(P \cap Q))) \subseteq n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(P))) \cap n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(Q))) = n\text{-}int(P) \cap n\text{-}int(Q)$ (by guess) $= n\text{-}int(P \cap Q)$. Then $n\text{-}int(P \cap Q) = n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(P \cap Q)))$ and hence $P \cap Q$ is $t^{\#}_{\alpha}$-nI-set.

Remark 2.6. *An ideal nanotopological space the notions of* β*-*nI*-open sets and* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-sets are independent.*

Example 2.5. *In Example 2.1,*

1. $A = \{m_1\}$ *is* β*-*nI*-open set but not* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set.*
2. $B = \{m_2\}$ *is* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set but not* β*-*nI*-open.*

Theorem 2.2. *For a subset* A *of a space* $(U, \mathcal{N}, I)$, *the following properties are equivalent:*

1. A *is* n*-open;*
2. A *is* β*-*nI*-open and a* $\mathcal{R}^{\#}_{\alpha}$*-*$nI$*-set.*

Proof.

(1)⇒(2): (2) follows by Remark 3.3 of [7] and (1) of Remark 2.4.

(2)⇒(1): Given A is a $\mathcal{R}^{\#}_{\alpha}$-$nI$-set. So $A = P \cap Q$ where P is n-open and Q is $t^{\#}_{\alpha}$-nI-set. Then $A \subseteq P = n\text{-}int(P)$. Also A is β-nI-open implies $A \subseteq n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(A))) \subseteq n\text{-}cl^{\star}(n\text{-}int(n\text{-}cl^{\star}(Q))) = n\text{-}int(Q)$ since Q is $t^{\#}_{\alpha}$-nI-set. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$ and hence A is n-open.

Remark 2.7. *For a subset* A *of a space* $(U, \mathcal{N}, I)$, *the following relations are true:*

1. A *is* n*-open* $\Rightarrow$ A *is strong* $\mathcal{R}$*-*nI*-set.*
2. A *is t-*nI*-set* A *with* $n\text{-}int(n\text{-}cl^{\star}(A)) = n\text{-}cl^{\star}(n\text{-}int(A))$ $\Rightarrow$ A *is strong* $\mathcal{R}^{\#}$*-*nI*-set.*

Proof.

Proof follows straight, since the Definition of strong $\mathcal{R}$-nI-set.

Remark 2.8. *The converses of Remark 2.7(1) is not true as shown in the next Example.*

Example 2.6. *In Example 2.1, $A = \{m_2\}$ is strong $\mathcal{R}$-nI-set but not n-open.*

Proposition 2.3. *In a space $(U, \mathcal{N}, I)$, each strong $\mathcal{R}$-nI-set is a $\mathcal{R}$-nI-set.*

Proof.

Proof follows from the fact that t-nI-set A with $n\text{-}int(n\text{-}cl^\star(A)) = n\text{-}cl^\star(n\text{-}int(A))$ is t-nI-set, which is a $\mathcal{R}$-nI-set by (3) Definition 3.1 of [8].

Remark 2.9. *The converse of the Proposition 2.3 is need not be true as shown in the next Examples.*

Example 2.7. *In Example 2.1, $A = \{m_1, m_2, m_3\}$ is $\mathcal{R}$-nI-set but not strong $\mathcal{R}$-nI-set.*

Theorem 2.3. *For a subset A of a space $(U, \mathcal{N}, I)$, the following properties are equivalent:*

1. *A is n-open;*
2. *A is b-nI-open and strong $\mathcal{R}$-nI-set.*

Proof.

(1)$\Rightarrow$(2): (2) follows by Remark 3.3 of [7] and (1) of Remark 2.7.

(2)$\Rightarrow$(1): Given A is strong $\mathcal{R}$-nI-set. So $A = P \cap Q$ where P is n-open and Q is t-nI-set with $n\text{-}int(n\text{-}cl^\star(Q)) = n\text{-}cl^\star(n\text{-}int(Q))$. Then $A \subseteq P = n\text{-}int(P)$. Also A is b-nI-open implies $A \subseteq n\text{-}int(n\text{-}cl^\star(A)) \cup n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}int(n\text{-}cl^\star(Q)) \cup n\text{-}cl^\star(n\text{-}int(Q)) = n\text{-}int(Q)$ by guess. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$ and hence A is n-open.

Remark 2.10. *An ideal nanotopological space the notions of b-nI-open sets and nI-open strong $\mathcal{R}$-nI-sets are independent.*

Example 2.8. *In Example 2.1,*

1. *$A = \{m_2\}$ is strong $\mathcal{R}$-nI-set but not b-nI-open.*
2. *$B = \{m_3\}$ is b-nI-open but not strong $\mathcal{R}^\#$-nI-set*

3. Several nano regular sets in an ideal spaces

Definition 3.1. A subset A of an ideal nanotopological space $(U, \mathcal{N}, I)$ is called a

1. nano semi-I-regular (briefly semi-nI-regular) if A is semi-nI-open and t-nI-set.
2. nano pre-I-regular (briefly pre-nI-regular) if A is pre-nI-open and $t^\#$-nI-set.
3. nano β-I-regular (briefly β-nI-regular) if A is β-nI-open and t_α-nI-set.
4. nano $\mathcal{O}_s$-I-set (briefly $\mathcal{O}_s$-nI-set) if $A = P \cap Q$ where P is n-open and Q is semi-nI-regular.
5. nano $\mathcal{O}_p$-I-set if (briefly $\mathcal{O}_p$-nI-set) $A = P \cap Q$ where P is n-open and Q is pre-nI-regular.
6. nano $\mathcal{O}_\beta$-I-set if (briefly $\mathcal{O}_\beta$-nI-set) $A = P \cap Q$, where P is n-open and Q is β-nI-regular.

Example 3.1. *In Example 2.1,*

1. *$A = \{m_1, m_3\}$ is semi-nI-regular.*

2. $B = \{m_4\}$ *is pre-nI-regular, β-nI-regular, $\mathcal{O}_s$-nI-set, $\mathcal{O}_p$-nI-set and $\mathcal{O}_\beta$-nI-set.*

Remark 3.1. *An ideal nanotopological space $(U, \mathcal{N}, I)$,*

1. *each semi-nI-regular set is semi-nI-open.*
2. *each semi-nI-regular set is a t-nI-set.*

Remark 3.2. *The converse in every part of the Remark 3.1 is need not be true as shown in the next two Examples.*

Example 3.2. *In Example 2.1,*

1. $A = \{m_1, m_3, m_4\}$ *is semi-nI-open but not semi-nI-regular.*
2. $B = \{m_2\}$ *is t-nI-set but not semi-nI-regular.*

Theorem 3.1. *A subset A of an ideal nanotopological space $(U, \mathcal{N}, I)$ is semi-nI-regular $\iff$ it is both β-nI-open and semi-nI-closed.*

Proof.

If A is semi-nI-regular, then A is both semi-nI-open and a t-nI-set. Since A is semi-nI-open, A is β-nI-open by Theorem 3.12 [3]. Also A is t-nI-set by assumption. Hence by Theorem 3.25 [3], A is semi-nI-closed.

Conversely, let A be semi-nI-closed and β-nI-open. Since A is semi-nI-closed, by Theorem 3.25 [3], A is t-nI-set. Since A is β-nI-open, $A \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A))) = n\text{-}cl^\star(n\text{-}int(A))$. Therefore A is semi-nI-open. Since A is both semi-nI-open and t-nI-set, A is semi-nI-regular.

Remark 3.3. *An ideal nanotopological space the notions of semi-nI-closed sets and β-nI-open sets are independent.*

Example 3.3. *In Example 2.1,*

1. $A = \{m_2\}$ *is semi-nI-closed but not β-nI-open.*
2. $B = \{m_1\}$ *is β-nI-open but not semi-nI-closed.*

Remark 3.4. *An ideal nanotopological space $(U, \mathcal{N}, I)$,*

1. *each semi-nI-regular set is semi-nI-open.*
2. *each semi-nI-regular set is a t-nI-set.*

Remark 3.5. *The converse in every part of the Remark 3.4 is need not be true as shown in the following two Examples.*

Example 3.4. *In Example 2.1,*

1. $A = \{m_4\}$ *is semi-nI-open but not semi-nI-regular.*
2. $B = \{m_2, m_4\}$ *is t-nI-set but not semi-nI-regular.*

Proposition 3.1. *An ideal nanotopological space $(U, \mathcal{N}, I)$,*

1. *each n-open set is $\mathcal{O}_s$-nI-set.*

2. *each semi-nI-regular set is $\mathcal{O}_s$-nI-set.*

Proof.

This is obvious from the Definition of $\mathcal{O}_s$-nI-set.

Remark 3.6. *The converse in every part of the Proposition 3.1 is need not be true as shown in the following two Examples.*

Example 3.5. *In Example 2.1,*

1. $A = \{m_1, m_2, m_3\}$ *is $\mathcal{O}_s$-nI-set but not n-open.*
2. $B = \{m_1, m_3, m_4\}$ *is $\mathcal{O}_s$-nI-set but not semi-nI-regular.*

Theorem 3.2. *For a subset A of a space $(U, \mathcal{N}, I)$, the following properties are equivalent:*

1. *A is n-open,*
2. *A is pre-nI-open and $\mathcal{O}_s$-nI-set.*

Proof.

(1)$\Rightarrow$(2): This is obvious by Remark 3.3 of [7] and (1) of Proposition 3.1.

(2)$\Rightarrow$(1): Given A is $\mathcal{O}_s$-nI-set. So, $A = P \cap Q$ where P is n-open and Q is semi-nI-regular. Then $A \subseteq P = n\text{-}int(P)$. Also A is pre-nI-open implies $A \subseteq n\text{-}int(n\text{-}cl^\star(A)) \subseteq n\text{-}int(n\text{-}cl^\star(Q)) = n\text{-}int(Q)$ for Q is t-nI-set being semi-nI-regular. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$ and hence A is n-open.

Remark 3.7. *An ideal nanotopological space the notions of $\mathcal{O}_s$-nI-sets and pre-nI-open sets are independent.*

Example 3.6. *In Example 2.1,*

1. $A = \{m_1, m_2, m_3\}$ *is $\mathcal{O}_s$-nI-set but not pre-nI-open.*
2. $B = \{m_1, m_4\}$ *is pre-nI-open but not $\mathcal{O}_s$-nI-set.*

Proposition 3.2. *In a space $(U, \mathcal{N}, I)$,*

1. *each pre-nI-regular set is pre-nI-open.*
2. *each pre-nI-regular set is $t^\#$-nI-set.*

Remark 3.8. *The converse in every part of the Proposition 3.2 is need not be true as shown in the next two Examples.*

Example 3.7. *In Example 2.1,*

1. $A = \{m_1, m_3\}$ *is pre-nI-open but not pre-nI-regular.*
2. $B = \{m_2\}$ *is $t^\#$-nI-set but not pre-nI-regular.*

Proposition 3.3. *In a space $(U, \mathcal{N}, I)$,*

1. *each n-open set is $\mathcal{O}_p$-nI-set.*
2. *each pre-nI-regular set is $\mathcal{O}_p$-nI-set.*

Proof.

Proof follows straight from the Definition of a $\mathcal{O}_p$-nI-set.

Remark 3.9. *The converse in every part of the Proposition 3.3 is need not be true as shown in the following two Examples.*

Example 3.8. *In Example 2.1,*

1. $A = \{m_1\}$ *is* $\mathcal{O}_p$-nI*-set but not* n*-open.*
2. $B = \{m_1, m_3, m_4\}$ *is* $\mathcal{O}_p$-nI*-set but not pre-*nI*-regular.*

Theorem 3.3. *For a subset* A *of a space* $(U, \mathcal{N}, I)$*, the following properties are equivalent:*

1. A *is* n*-open;*
2. A *is semi-*nI*-open and* $\mathcal{O}_p$-nI*-set.*

Proof. . (1)⇒(2): (2) follows from Remark 3.3 of [7] and (1) of Proposition 3.3.

(2)⇒(1): Given A is $\mathcal{O}_p$-nI-set. So, $A = P \cap Q$ where P is n-open and Q is pre-nI-regular. Then $A \subseteq P = n\text{-}int(P)$. Also A is semi-nI-open implies $A \subseteq n\text{-}cl^\star(n\text{-}int(A)) \subseteq n\text{-}cl^\star(n\text{-}int(Q)) = n\text{-}int(Q)$ since Q is $t^\#$-nI-set being pre-nI-regular. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$ and hence A is n-open.

Proposition 3.4. *An ideal nanotopological space* $(U, \mathcal{N}, I)$*, each t-*nI*-set is* t_α-nI*-set.*

Proof. Let A be t-nI-set. Then $n\text{-}int(A) = n\text{-}int(n\text{-}cl^\star(A))$. Now $n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subseteq n\text{-}int(n\text{-}cl^\star(A)) = n\text{-}int(A)$. Also $n\text{-}int(A) \subseteq n\text{-}int(A) \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$. Thus $n\text{-}int(A) = n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$ and so A is t_α-nI-set.

Remark 3.10. *The converse of Proposition 3.4 is not true as shown in the next Example.*

Example 3.9. *In Example 2.1,* $A = \{m_1, m_2\}$ *is* t_α-nI*-set but not t-*nI*-set.*

Proposition 3.5. *In a space* $(U, \mathcal{N}, I)$*, each* $t^\#$-nI*-set is* t_α-nI*-set.*

Proof.

Let A be a $t^\#$-nI-set. Then $n\text{-}int(A) = n\text{-}cl^\star(n\text{-}int(A))$. Now $n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subseteq n\text{-}cl^\star(n\text{-}int(A)) = n\text{-}int(A)$. Also, $n\text{-}int(A) \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$. Thus $n\text{-}int(A) = n\text{-}int(n\text{-}cl^\star(n\text{-}int(A)))$ and so A is t_α-nI-set.

Remark 3.11. *The converse of Proposition 3.5 is not true as shown in the next Example.*

Example 3.10. *In Example 2.1,* $A = \{m_1, m_3\}$ *is* t_α-nI*-set but not* $t^\#$-nI*-set.*

Proposition 3.6. *In a space* $(U, \mathcal{N}, I)$*,*

1. *each semi-*nI*-regular set is* β-nI*-regular.*
2. *each pre-*nI*-regular set is* β-nI*-regular.*
3. *each* β-nI*-regular set is* β-nI*-open.*
4. *each* β-nI*-regular set is* t_α-nI*-set.*

Proof.

(1). Proof follows from the fact that each semi-nI-open set is β-nI-open by Remark 3.3 of [7] and each t-nI-set is t_α-nI-set by Proposition 3.4.

(2). Proof follows from the fact that each pre-nI-open set is β-nI-open by Remark 3.3 of [7] and each $t^{\#}$-nI-set is t_α-nI-set by Proposition 3.5.

(3), (4). The proofs follow from their Definitions.

Remark 3.12. *The converse in every part of the Proposition 3.6 is need not be true as shown in the next three Examples.*

Example 3.11. *In Example 2.1,*

1. $A = \{m_1\}$ *is* β-nI*-regular set but not semi-*nI*-regular.*
2. $B = \{m_1, m_3, m_4\}$ *is* β-nI*-open set but not* β-nI*-regular.*
3. $C = \{m_2\}$ *is* t_α-nI*-set but not* β-nI*-regular.*

Proposition 3.7. *An ideal nanotopological space* $(U, \mathcal{N}, I)$*,*

1. *each* n*-open set is a* $\mathcal{O}_\beta$-nI*-set.*
2. *each* β-nI*-regular set is* $\mathcal{O}_\beta$-nI*-set.*

Remark 3.13. *The converse in every part of Proposition 3.7(2) is need not be true as shown in the next Examples.*

Example 3.12. *In Example 2.1,* $A = \{m_1, m_3, m_4\}$ *is* $\mathcal{O}_\beta$-nI*-set but not* β-nI*-regular.*

Proposition 3.8. *In a space* $(U, \mathcal{N}, I)$*, each* $\mathcal{O}_\beta$-nI*-set is* β-nI*-open.*

Proof.

Let A be a $\mathcal{O}_\beta$-nI-set. Then $A = P \cap Q$, where P is n-open and Q is β-nI-regular. Hence Q is β-nI-open. So $A = P \cap Q \subseteq P \cap n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(Q))) \subseteq n\text{-}cl^\star(P \cap n\text{-}int(n\text{-}cl^\star(Q)))$ by Lemma 3.23 of [7]. And $n\text{-}cl^\star(P \cap n\text{-}int(n\text{-}cl^\star(Q))) = n\text{-}cl^\star(n\text{-}int(P) \cap n\text{-}int(n\text{-}cl^\star(Q))) = n\text{-}cl^\star(n\text{-}int(P \cap n\text{-}cl^\star(Q))) \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(P \cap Q))) = n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$. Thus $A \subseteq n\text{-}cl^\star(n\text{-}int(n\text{-}cl^\star(A)))$. Hence A is β-nI-open.

Theorem 3.4. *For a subset* A *of a space* $(U, \mathcal{N}, I)$*, the following properties are equivalent:*

1. A *is* β-nI*-regular.*
2. A *is a* t_α-nI*-set and* $\mathcal{O}_\beta$-nI*-set.*

Proof.

(1)$\Rightarrow$(2): Proof follows directly since each β-nI-regular set is t_α-nI-set by Proposition 3.6 (4) and $\mathcal{O}_\beta$-nI-set by (2) of Proposition 3.7.

(2)$\Rightarrow$(1): Let A be t_α-nI-set and $\mathcal{O}_\beta$-nI-set. Since A is $\mathcal{O}_\beta$-nI-set, by Proposition 3.8 A is β-nI-open. Thus A is t_α-nI-set as well as β-nI-open set. Hence A is β-nI-regular.

Remark 3.14. *An ideal nanotopological space the notions of* t_α-nI*-sets and* $\mathcal{O}_\beta$-nI*-sets are independent.*

Example 3.13. *In Example 2.1,*

1. $A = \{m_1, m_3, m_4\}$ *is* $\mathcal{O}_\beta$*-*nI*-set but not* t_α*-*nI*-set.*
2. $B = \{m_2\}$ *is* t_α*-*nI*-set but not* $\mathcal{O}_\beta$*-*nI*-set.*

Theorem 3.5. *For a subset* A *of a space* $(U, \mathcal{N}, I)$*, the following properties are equivalent:*

1. A *is* n*-open,*
2. A *is* α*-*nI*-open and* $\mathcal{O}_\beta$*-*nI*-set.*

Proof.

(1)$\Rightarrow$(2): Proof follows directly by Remark 3.3 of [7] and by (1) of Proposition 3.7.

(2)$\Rightarrow$(1): Given A is $\mathcal{O}_\beta$-nI-set. So, $A = P \cap Q$ where P is n-open and Q is β-nI-regular. Then $A \subseteq P = n\text{-}int(P)$. Also A is α-nI-open implies $A \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(A))) \subseteq n\text{-}int(n\text{-}cl^\star(n\text{-}int(Q))) = n\text{-}int(Q)$ for Q is t_α-nI-set being β-nI-regular. Thus $A \subseteq n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(A)$ and hence A is n-open.

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