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Assignment problem with neutrosophic costs and its solution methodology

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Abstract: In this study, a single valued neutrosophic number has been presented in a new direction so that a decision maker has a scope of flexibility to choose different numbers in their study. Its structural characteristics are also studied here. Then this kind of number has been converted into a numeric value by means of a parameter (whose value is pre-assigned) to practice in real fields and using this, a ranking function is defined to compare two or more single valued neutrosophic numbers. In continuation, an assignment problem and its solution methodology have been developed in neutrosophic environment. Two real problems are solved to demonstrate the proposed method.

Key words: Neutrosophic set, Single valued neutrosophic number, Ranking function, Assignment problem.

1. Introduction

Decision makers will have to go through an uncertain atmosphere in some cases whenever taking a decision in real world problems with incomplete and imprecise information since our daily life activities are being complicated from day to day. Classical set theory is not suitable to handle that situation as it only indicates whether an element either belongs or not to a set. Although the probability theory was an age old tool but the theory of fuzzy sets [15], the theory of intuitionistic fuzzy sets [2] brought a nice opportunity in that concern. Fuzzy set theory provides the degree of belongingness of an element whereas intuitionistic fuzzy set theory gives the degree of belongingness as well as the degree of not belongingness of an element. Both the theories are widely practiced in vague and uncertain atmosphere from their initiation.

To deal with uncertainty more precisely, Smarandache [11, 12] generalised the intuitionistic fuzzy set to neutrosophic set (NS). In neutrosophic logic, each object is characterized by a triplet (T, I, F) where T, I, F respectively refer the truth-membership value, the indeterminacy-membership value and the falsity-membership value. Intuitionistic fuzzy set theory can not provide the indeterminate information of an object. In NS theory $T, I, F \in]^{-}0, 1^{+}[$ and they are independent each. Accordingly, the limitation '$0 \leq$ membership value + non-membership value ≤ 1' in intuitionistic fuzzy set theory was replaced by '$^{-}0 \leq \sup T + \sup I + \sup F \leq 3^{+}$' in NS theory. In order to practice the NS theory in real field, Wang et al. [13] brought the concept of single valued neutrosophic set where $T, I, F \in [0, 1]$ only.

The notion of ranking of fuzzy numbers and intuitionistic fuzzy numbers is being widely practiced in decision making and optimization theory over a last few decades. Chen [4] proposed a fuzzy assignment model and proved some related theorems. Lin and Wen [7] solved an assignment problem with fuzzy interval number costs. Deli and Subas [5] have presented a ranking method of neutrosophic number and applied it to multi-attributive decision making problems. Some different kinds of ranking technique [6, 8–10, 14] in decision making are re-

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ported here.
In the present paper the concept of single valued neutrosophic number (SVN-number) has been introduced in a different mode along with the study of its structural characteristics. Then a model of assignment problem with its solution methodology have been developed in neutrosophic environment. The proposed work are also demonstrated by two real problems. Organisation of this paper is as follows.
Some preliminary useful definitions are placed in Section 2. The concept of SVN-number has been introduced in a different way in Section 3. In Section 4, a model of assignment problem in neutrosophic environment has been presented with proper demonstration by two real problems. Finally, the conclusion of the present work has been drawn in Section 5.

2. Preliminaries

We recall some necessary definitions and results to make out the main thought.

Definition 2.1. [11] An NS B over the universe U is characterized by a triplet (T_B, I_B, F_B) respectively called truth-membership function, indeterminacy-membership function and falsity-membership function where T_B, I_B, F_B are real standard or nonstandard subsets of $]^-0, 1^+[$ i.e., $T_B, I_B, F_B : U \to]^-0, 1^+[$. Here $1^+ = 1+\epsilon$, where 1 is its standard part and ϵ is its non-standard part. Similarly $^-0 = 0 - \epsilon$, where 0 is its standard part and ϵ is its non-standard part. Thus the NS B is defined as : $B = \{< u, T_B(u), I_B(u), F_B(u) >: u \in U\}$ with $^-0 \leq \sup T_B(u) + \sup I_B(u) + \sup F_B(u) \leq 3^+$.

Definition 2.2. [13] In an NS B over U, if the components $T_B(u)$, $I_B(u)$ and $F_B(u)$ are all real standard elements of $[0, 1]$ for $u \in U$, then it is called a single valued neutrosophic set. Thus it is defined as : $B = \{< u, T_B(u), I_B(u), F_B(u) >: u \in U\}$ with $T_B(u), I_B(u), F_B(u) \in [0, 1]$ and $0 \leq \sup T_B(u)+\sup I_B(u)+\sup F_B(u) \leq 3$.

Definition 2.3. [3] Let B be an NS over the universal set U. The (α, β, γ) -cut of U is denoted by $B_{(\alpha,\beta,\gamma)}$ and is defined as : $B_{(\alpha,\beta,\gamma)} = \{u \in U : T_B(u) \geq \alpha, I_B(u) \leq \beta, F_B(u) \leq \gamma\}$ where $\alpha, \beta, \gamma \in [0, 1]$ with $0 \leq \alpha + \beta + \gamma \leq 3$. Clearly, it is a crisp subset U.

Definition 2.4. [1]A fuzzy number A consists of a pair (A_L, A_R) and satisfies the followings :
(i) A_L is left continuous function and bounded monotone increasing.
(ii) A_R is right continuous function and bounded monotone decreasing.
(iii) $A_L(r) \leq A_R(r)$, $r \in [0, 1]$.

A trapezoidal fuzzy number B is expressed as (a, b, α, β) where $[a, b]$ is interval defuzzifier and $\alpha(> 0), \beta(> 0)$ are respectively called the left fuzziness, right fuzziness. The support of B is $(a-\alpha, b+\beta)$ and its membership function is :

$$B(x) = \begin{cases} \frac{1}{\alpha}(x - a + \alpha), & x \in [a - \alpha, a], \\ 1, & x \in [a, b], \\ \frac{1}{\beta}(b - x + \beta), & x \in [b, b + \beta], \\ 0, & \text{otherwise.} \end{cases}$$

In parametric form $B_L(r) = a - \alpha + \alpha r$, $B_R(r) = b + \beta - \beta r$.
For arbitrary trapezoidal fuzzy numbers $A = (A_L, A_R), B = (B_L, B_R)$ and scalar $k > 0$, the addition and scalar multiplication are defined by :
$(A + B)_L(r) = A_L(r) + B_L(r), (A + B)_R(r) = A_R(r) + B_R(r)$ and $(kA)_L(r) = kA_L(r), (kA)_R(r) = kA_R(r)$.

3. Single valued neutrosophic number

In this section, the concept of SVN-number is presented in a new direction along with the study of its characteristics. Then a ranking function has been constructed in order to compare the SVN-numbers.

Definition 3.1. An SVN-number $\tilde{a} = \langle [a_1, b_1, \sigma_1, \eta_1],\ [a_2, b_2, \sigma_2, \eta_2], [a_3, b_3, \sigma_3, \eta_3] \rangle$ is one kind of NS defined over the set of real numbers $\mathbf{R}$ where $\sigma_i(>0), \eta_i(>0)$ are respectively the left spreads, the right spreads and $[a_i, b_i]$ are the modal intervals of truth-membership, indeterminacy-membership and the falsity-membership functions for $i = 1, 2, 3$, respectively in $\tilde{a}$. The truth-membership, indeterminacy-membership and the falsity-membership functions are defined as follows :

$$T_{\tilde{a}}(x) = \begin{cases} \frac{1}{\sigma_1}(x - a_1 + \sigma_1), & a_1 - \sigma_1 \le x \le a_1, \\ 1, & x \in [a_1, b_1], \\ \frac{1}{\eta_1}(b_1 - x + \eta_1), & b_1 \le x \le b_1 + \eta_1, \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{1}{\sigma_2}(a_2 - x), & a_2 - \sigma_2 \le x \le a_2, \\ 0, & x \in [a_2, b_2], \\ \frac{1}{\eta_2}(x - b_2), & b_2 \le x \le b_2 + \eta_2, \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{1}{\sigma_3}(a_3 - x), & a_3 - \sigma_3 \le x \le a_3, \\ 0, & x \in [a_3, b_3], \\ \frac{1}{\eta_3}(x - b_3), & b_3 \le x \le b_3 + \eta_3, \\ 1, & \text{otherwise.} \end{cases}$$

In parametric form, an SVN-number $\tilde{a}$ consists of three pairs $(T^l_{\tilde{a}}, T^u_{\tilde{a}}), (I^l_{\tilde{a}}, I^u_{\tilde{a}}), (F^l_{\tilde{a}}, F^u_{\tilde{a}})$ of functions $T^l_{\tilde{a}}(r), T^u_{\tilde{a}}(r)$, $I^l_{\tilde{a}}(r), I^u_{\tilde{a}}(r), F^l_{\tilde{a}}(r), F^u_{\tilde{a}}(r), r \in [0, 1]$ and satisfies the following requirements.

(i) $T^l_{\tilde{a}}, I^u_{\tilde{a}}, F^u_{\tilde{a}}$ are continuous function and bounded monotone increasing.

(ii) $T^u_{\tilde{a}}, I^l_{\tilde{a}}, F^l_{\tilde{a}}$ are continuous function and bounded monotone decreasing.

(iii) $T^l_{\tilde{a}}(r) \le T^u_{\tilde{a}}(r), I^l_{\tilde{a}}(r) \ge I^u_{\tilde{a}}(r), F^l_{\tilde{a}}(r) \ge F^u_{\tilde{a}}(r)$.

with $T^l_{\tilde{a}}(r) = a_1 - \sigma_1 + \sigma_1 r$, $T^u_{\tilde{a}}(r) = b_1 + \eta_1 - \eta_1 r$; $I^l_{\tilde{a}}(r) = a_2 - \sigma_2 r$, $I^u_{\tilde{a}}(r) = b_2 + \eta_2 r$ and $F^l_{\tilde{a}}(r) = a_3 - \sigma_3 r$, $F^u_{\tilde{a}}(r) = b_3 + \eta_3 r$.

Definition 3.2. An SVN-number $\tilde{a}$ is called an SVTN-number if three modal intervals in $\tilde{a}$ are equal. Thus $\tilde{c} = \langle [a_0, b_0, \sigma_1, \eta_1], [a_0, b_0, \sigma_2, \eta_2], [a_0, b_0, \sigma_3, \eta_3] \rangle$ is an SVTN-number.

Let $\tilde{a} = \langle [a, a', \sigma_1, \eta_1], [a, a', \sigma_2, \eta_2], [a, a', \sigma_3, \eta_3] \rangle$ and $\tilde{b} = \langle [b, b', \xi_1, \delta_1], [b, b', \xi_2, \delta_2], [b, b', \xi_3, \delta_3] \rangle$ be two SVTN-numbers. Then for any real number x,

(i) Image of $\tilde{a}$:

$$-\tilde{a} = \langle [-a', -a, \eta_1, \sigma_1], [-a', -a, \eta_2, \sigma_2], [-a', -a, \eta_3, \sigma_3] \rangle.$$

(ii) Addition :

$$\tilde{a} + \tilde{b} = \langle [a + b, a' + b', \sigma_1 + \xi_1, \eta_1 + \delta_1], [a + b, a' + b', \sigma_2 + \xi_2, \eta_2 + \delta_2], [a + b, a' + b', \sigma_3 + \xi_3, \eta_3 + \delta_3] \rangle.$$

(iii) Scalar multiplication :

$$x\tilde{a} = \langle ([xa, xa', x\sigma_1, x\eta_1],\ [xa, xa', x\sigma_2, x\eta_2], [xa, xa', x\sigma_3, x\eta_3] \rangle \quad \text{for } x > 0.$$

$$x\tilde{a} = \langle [xa', xa, -x\eta_1, -x\sigma_1], [xa', xa, -x\eta_2, -x\sigma_2], [xa', xa, -x\eta_3, -x\sigma_3] \rangle \quad \text{for } x < 0.$$

Definition 3.3. An SVTN-number $\tilde{a}$ is called an SVTrN-number if the modal interval in $\tilde{a}$ is reduced to a modal point i.e., if the end points of the modal interval are equal. Thus $\tilde{a} = \langle [a_0, \sigma_1, \eta_1]\ \ [a_0, \sigma_2, \eta_2],\ [a_0, \sigma_3, \eta_3] \rangle$ is an SVTrN-number.

Remark 3.1. *(Graphical presentation of SVN-numbers)*
By Definition 3.2, we consider different support (i.e. bases of trapeziums (triangles) formed) for T, I, F. Thus, the supports and heights are allowed together to differ the value of T, I, F in the present study. Then decision maker has a scope of flexibility to choose and compare different SVN-numbers in their study. The fact is shown by the graphical figures.

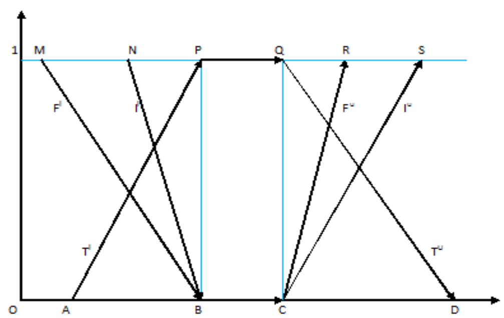

Figure: SVTN-number

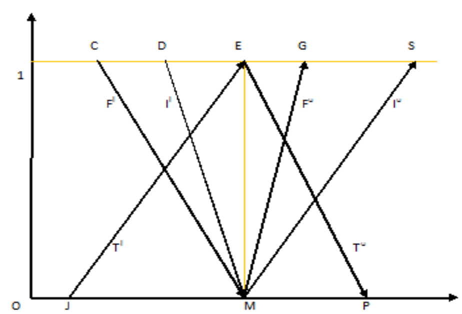

Figure : SVTrN-number

Definition 3.4. 1. The zero SVTN-number is denoted by $\tilde{0}$ and is defined as :

$$\tilde{0} = \langle [0,0,0,0],\ [0,0,0,0], ([0,0,0,0] \rangle .$$

2. The zero SVTrN-number is denoted by $\tilde{0}$ and is defined as :

$$\tilde{0} = \langle [0,0,0],\ [0,0,0], [0,0,0] \rangle .$$

Definition 3.5. The values of each component corresponding an SVN-number $\tilde{a} = \langle [a_1, b_1, \sigma_1, \eta_1], [a_2, b_2, \sigma_2, \eta_2], [a_3, b_3, \sigma_3, \eta_3] \rangle$ is calculated here using cut set.
1. Any α- cut set of the SVN-number $\tilde{a}$ for truth membership function is denoted by $\tilde{a}_\alpha$ and is given by a closed interval as : $\tilde{a}_\alpha = [a_1 - \sigma_1 + \sigma_1\alpha, b_1 + \eta_1 - \eta_1\alpha]$ for $\alpha \in [0,1]$.
The value of $\tilde{a}$ for α- cut set is denoted by $V_T(\tilde{a})$ and is calculated as :

$$V_T(\tilde{a}) = \int_0^1 [(a_1 - \sigma_1 + \sigma_1\alpha) + (b_1 + \eta_1 - \eta_1\alpha)] f(\alpha)\, d\alpha = \frac{1}{6}(3a_1 + 3b_1 - \sigma_1 + \eta_1), \quad \text{taking } f(\alpha) = \alpha.$$

where $f(\alpha) \in [0,1], f(0) = 0$ and $f(\alpha)$ is monotone increasing for $\alpha \in [0,1]$.

2. Any β- cut set of the SVN-number $\tilde{a}$ for indeterminacy membership function is denoted by $\tilde{a}^\beta$ and is given by a closed interval as : $\tilde{a}^\beta = [a_2 - \sigma_2\beta, b_2 + \eta_2\beta]$ for $\beta \in [0,1]$.
The value of $\tilde{a}$ for β- cut set is denoted by $V_I(\tilde{a})$ and is calculated as :

$$V_I(\tilde{a}) = \int_0^1 [(a_2 - \sigma_2\beta) + (b_2 + \eta_2\beta)] g(\beta)\, d\beta = \frac{1}{6}(3a_2 + 3b_2 - \sigma_2 + \eta_2), \quad \text{taking } g(\beta) = 1 - \beta.$$

where $g(\beta) \in [0,1], g(1) = 0$ and $g(\beta)$ is monotone decreasing for $\beta \in [0,1]$.

3. Any γ- cut set of the SVN-number $\tilde{a}$ for falsity membership function is denoted by ${}^{\gamma}\tilde{a}$ and is given by a closed interval as : ${}^{\gamma}\tilde{a} = [a_3 - \sigma_3\gamma, b_3 + \eta_3\gamma]$ for $\gamma \in [0,1]$.
The value of $\tilde{a}$ for γ- cut set is denoted by $V_F(\tilde{a})$ and is calculated as :

$$V_F(\tilde{a}) = \int_0^1 [(a_3 - \sigma_3\gamma) + (b_3 + \eta_3\gamma)]h(\gamma)\, d\gamma = \frac{1}{6}(3a_3 + 3b_3 - \sigma_3 + \eta_3), \quad \text{taking } h(\gamma) = 1 - \gamma.$$

where $h(\gamma) \in [0,1], h(1) = 0$ and $h(\gamma)$ is monotone decreasing for $\gamma \in [0,1]$.

Definition 3.6. For $\theta \in [0,1]$, the θ- weighted value of an SVN-number $\tilde{b}$ is denoted by $V_\theta(\tilde{b})$ and is defined as :

$V_\theta(\tilde{b}) = \theta^n V_T(\tilde{b}) + (1-\theta^n)V_I(\tilde{b}) + (1-\theta^n)V_F(\tilde{b})$, n being any natural number.

Thus, the θ-weighted value for the SVN-number $\tilde{a}$ stated in Definition 3.5 is :

$V_\theta(\tilde{a}) = \frac{1}{6}[(3a_1 + 3b_1 - \sigma_1 + \eta_1)\theta^n + (3a_2 + 3b_2 - \sigma_2 + \eta_2)(1-\theta^n) + (3a_3 + 3b_3 - \sigma_3 + \eta_3)(1-\theta^n)$.

Corollary 3.1. *The θ - weighted value obeys the following disciplines for two SVN-numbers $\tilde{a}, \tilde{b}$.*

(i) $V_\theta(\tilde{a} \pm \tilde{b}) = V_\theta(\tilde{a}) \pm V_\theta(\tilde{b})$.
(ii) $V_\theta(\mu\tilde{a}) = \mu V_\theta(\tilde{a})$, *$\mu$ being any real number.*
(iii) $V_\theta(\tilde{a} - \tilde{a}) = V_\theta(\tilde{0})$.
(iv) $V_\theta(\tilde{a})$ *is monotone increasing or decreasing or constant according as* $V_T(\tilde{a}) > V_I(\tilde{a}) + V_F(\tilde{a})$ *or* $V_T(\tilde{a}) < V_I(\tilde{a}) + V_F(\tilde{a})$ *or* $V_T(\tilde{a}) = V_I(\tilde{a}) + V_F(\tilde{a})$ *respectively.*

Proof. We shall prove (iv) only. Remaining can be easily verified by taking two arbitrary SVN-numbers. Here,

$V_\theta(\tilde{a}) = \theta^n V_T(\tilde{a}) + (1-\theta^n)V_I(\tilde{a}) + (1-\theta^n)V_F(\tilde{a}) \Rightarrow \frac{dV_\theta(\tilde{a})}{d\theta} = n\theta^{n-1}[V_T(\tilde{a}) - (V_I(\tilde{a}) + V_F(\tilde{a}))]$

Since $\theta \in [0,1]$, so $\frac{dV_\theta(\tilde{a})}{d\theta} >, < 0$ when $[V_T(\tilde{a}) - (V_I(\tilde{a}) + V_F(\tilde{a}))] >, < 0$ respectively. For $\theta \in (0,1]$, we have $\frac{dV_\theta(\tilde{a})}{d\theta} = 0$ when $[V_T(\tilde{a}) - (V_I(\tilde{a}) + V_F(\tilde{a}))] = 0$. This clears the fact. □

Definition 3.7. Let $SVN(\mathbf{R})$ be the set of all SVN-numbers defined over $\mathbf{R}$. For $\theta \in [0,1]$, a mapping $\Re_\theta : SVN(\mathbf{R}) \longrightarrow \mathbf{R}$ is called a ranking function and it is defined as : $\Re_\theta(\tilde{a}) = V_\theta(\tilde{a})$ for $\tilde{a} \in SVN(\mathbf{R})$.

An useful method for comparing of SVN-numbers is by practice of ranking function. For $\tilde{a}, \tilde{b} \in SVN(\mathbf{R})$, their order is defined as follows :

$V_\theta(\tilde{a}) > V_\theta(\tilde{b}) \Leftrightarrow \tilde{a} >_{\Re_\theta} \tilde{b}, \quad V_\theta(\tilde{a}) < V_\theta(\tilde{b}) \Leftrightarrow \tilde{a} <_{\Re_\theta} \tilde{b}, \quad V_\theta(\tilde{a}) = V_\theta(\tilde{b}) \Leftrightarrow \tilde{a} =_{\Re_\theta} \tilde{b}$.

Corollary 3.2. *Consider two SVTN-numbers* $\tilde{c} = \langle [x, y, \sigma_1, \eta_1], [x, y, \sigma_2, \eta_2], [x, y, \sigma_3, \eta_3] \rangle$ *and* $\tilde{d} = \langle [p, q, \omega_1, \xi_1], [p, q, \omega_2, \xi_2], [p, q, \omega_3, \xi_3] \rangle$ *with* $[x, y] = [p, q]$. *Then* $\tilde{c} >_{\Re_\theta} \tilde{d}$ *iff the followings hold.*

(i) $(\eta_1 + \omega_1) > (\sigma_1 + \xi_1)$ *for* $\theta = 1$.
(ii) $(\eta_2 + \eta_3) + (\omega_2 + \omega_3) > (\sigma_2 + \sigma_3) + (\xi_2 + \xi_3)$ *for* $\theta = 0$.

Proof.

$$
\begin{aligned}
(i)\quad & \eta_1+\omega_1>\sigma_1+\xi_1 \Leftrightarrow (\eta_1-\sigma_1)>(\xi_1-\omega_1)\\
\Leftrightarrow\quad & (\eta_1-\sigma_1)\theta^n>(\xi_1-\omega_1)\theta^n \qquad (\text{as} \quad \theta=1)\\
\Leftrightarrow\quad & (\eta_1-\sigma_1)\theta^n+\{(\eta_2-\sigma_2)+(\eta_3-\sigma_3)\}(1-\theta^n)\\
& \qquad >(\xi_1-\omega_1)\theta^n+\{(\xi_2-\omega_2)+(\xi_3-\omega_3)\}(1-\theta^n) \qquad (\text{as} \quad \theta=1)\\
\Leftrightarrow\quad & \frac{1}{6}[(3x+3y+\eta_1-\sigma_1)\theta^n+\{(3x+3y+\eta_2-\sigma_2)+(3x+3y+\eta_3-\sigma_3)\}(1-\theta^n)]\\
& >\frac{1}{6}[(3p+3q+\xi_1-\omega_1)\theta^n+\{(3p+3q+\xi_2-\omega_2)+(3p+3q+\xi_3-\omega_3)\}(1-\theta^n)]\\
& (\text{ as} \quad [x,y]=[p,q])\\
\Leftrightarrow\quad & V_\theta(\tilde{c})>V_\theta(\tilde{d}) \Leftrightarrow \tilde{c}>_{\Re_\theta}\tilde{d} \text{ with } [x,y]=[p,q]
\end{aligned}
$$

$$
\begin{aligned}
(ii)\quad & (\eta_2+\eta_3)+(\omega_2+\omega_3)>(\sigma_2+\sigma_3)+(\xi_2+\xi_3)\\
\Leftrightarrow\quad & \{(\eta_2-\sigma_2)+(\eta_3-\sigma_3)\}(1-\theta^n)>\{(\xi_2-\omega_2)+(\xi_3-\omega_3)\}(1-\theta^n) \quad (\text{as} \quad \theta=0)\\
\Leftrightarrow\quad & (\eta_1-\sigma_1)\theta^n+\{(\eta_2-\sigma_2)+(\eta_3-\sigma_3)\}(1-\theta^n)\\
& \qquad >(\xi_1-\omega_1)\theta^n+\{(\xi_2-\omega_2)+(\xi_3-\omega_3)\}(1-\theta^n) \qquad (\text{as} \quad \theta=0)\\
\Leftrightarrow\quad & \frac{1}{6}[(3x+3y+\eta_1-\sigma_1)\theta^n+\{(3x+3y+\eta_2-\sigma_2)+(3x+3y+\eta_3-\sigma_3)\}(1-\theta^n)]\\
& >\frac{1}{6}[(3p+3q+\xi_1-\omega_1)\theta^n+\{(3p+3q+\xi_2-\omega_2)+(3p+3q+\xi_3-\omega_3)\}(1-\theta^n)]\\
& (\text{ as} \quad [x,y]=[p,q])\\
\Leftrightarrow\quad & V_\theta(\tilde{c})>V_\theta(\tilde{d}) \Leftrightarrow \tilde{c}>_{\Re_\theta}\tilde{d} \text{ with } [x,y]=[p,q]
\end{aligned}
$$

Similar conclusion can be drawn in case of SVTrN-numbers. □

4. Assignment problem with neutrosophic costs

An assignment problem in crisp sense can be defined by an $n\times n$ cost matrix of real numbers as given in Table 1 which assigns men to jobs, jobs to machines etc. It is also assumed that one person can perform one job at a time and thus all the jobs will be assigned to all available persons and so on in other cases. The problem is optimal if it minimizes the total cost or minimizes the total time or maximizes the profit of performing all the jobs. where c_{ij} is the cost of assigning the j^{th} job to the i^{th} person. Mathematically, the problem can be put

Table 1. Tabular form of assignment problem.

	JOBS						
		J_1	J_2	J_3	$\cdots$	$\cdots$	J_n
PERSONS	P_1	c_{11}	c_{12}	c_{13}	$\cdots$	$\cdots$	c_{1n}
	P_2	c_{21}	c_{22}	c_{23}	$\cdots$	$\cdots$	c_{2n}
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	P_n	c_{n1}	c_{n2}	c_{n3}	$\cdots$	$\cdots$	c_{nn}

as :
Determine $x_{ij} \geq 0, \ i, j = 1, 2, \cdots, n$ which

$$\text{optimize} \qquad z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{such that} \qquad \sum_{j=1}^{n} x_{ij} = 1, \ 1 \leq i \leq n$$

$$\text{and} \qquad \sum_{i=1}^{n} x_{ij} = 1, \ 1 \leq j \leq n$$

$$\text{with} \qquad x_{ij} = \begin{cases} 1, & \text{if the } j^{th} \text{ job is assigned to the } i^{th} \text{ person} \\ 0, & \text{otherwise.} \end{cases}$$

Now if we consider the costs c_{ij} as SVN-numbers (we write $\tilde{a}_{ij}$), then the total cost $\tilde{z} =_{\Re_\theta} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} x_{ij}$ becomes an SVN-number. Then, we can not apply the crisp concept directly to optimize it. We shall adopt the following technique for that.

Proposition 4.1. *(Solution approach of assignment problem with neutrosophic costs)*
We now develop a method to find a solution of this class of problem. In the present study, the costs in assignment problem are taken as SVN-numbers. We shall first find the θ-weighted value function of each neutrosophic cost and thus we shall get a crisp cost matrix from a neutrosophic cost matrix for a pre-assigned θ. Now applying the computational procedure in crisp sense, we get optimal solutions of the assignment problem. Consequently the optimal value of the original problem can be calculated as : $\tilde{z} =_{\Re_\theta} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} x_{ij}$ which gives a numeric value using θ-weighted value function for that θ.

Definition 4.1. An SVN-number is said to be constant if it is θ independent after transforming it into a θ-weighted value function.
Thus $\tilde{b} = \langle [5, 8, 3, 43], [5, 8, 2, 2], [5, 8, 1, 2] \rangle$ is a constant SVTN-number as $V_\theta(\tilde{b}) = 13.17$ (approx) whatever the value of θ is.

Theorem 4.1. *If a constant SVN-number be added to any row and / or any column of the cost matrix of an assignment problem in neutrosophic environment, then the resulting assignment problem has the same optimal solution as the original problem.*

Proof. Let $[\tilde{a}_{ij}]$ be the cost matrix and suppose two constant SVN-numbers $\tilde{\alpha}_i$, $\tilde{\beta}_j$ be added to the i-th row and j-th column respectively for $1 \leq i, j \leq m$. Let $[\tilde{a}'_{ij}]$ be the new cost matrix where $\tilde{a}'_{ij} = \tilde{a}_{ij} + \tilde{\alpha}_i + \tilde{\beta}_j$ and the two objective functions be $\tilde{z} =_{\Re_\theta} \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}_{ij} x_{ij}$, $\tilde{z}' =_{\Re_\theta} \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij}$. Now,

$$\tilde{z}' =_{\Re_\theta} \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij} \ \Rightarrow \ V_\theta(\tilde{z}') = V_\theta(\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij})$$

$$\Rightarrow \quad V_\theta(\tilde{z}') = \sum_{i=1}^{m}\sum_{j=1}^{m} V_\theta(\tilde{a}'_{ij}x_{ij}) \quad \text{[by Corollary 3.1]}$$

$$\Rightarrow \quad V_\theta(\tilde{z}') = \sum_{i=1}^{m}\sum_{j=1}^{m} V_\theta[(\tilde{a}_{ij} + \tilde{\alpha}_i + \tilde{\beta}_j)x_{ij}]$$

$$\Rightarrow \quad V_\theta(\tilde{z}') = \sum_{i=1}^{m}\sum_{j=1}^{m} V_\theta(\tilde{a}_{ij}x_{ij}) + \sum_{i=1}^{m}\sum_{j=1}^{m} V_\theta(\tilde{\alpha}_i x_{ij}) + \sum_{i=1}^{m}\sum_{j=1}^{m} V_\theta(\tilde{\beta}_j x_{ij})$$

$$\Rightarrow \quad V_\theta(\tilde{z}') = V_\theta(\sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{a}_{ij}x_{ij}) + V_\theta(\sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{\alpha}_i x_{ij}) + V_\theta(\sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{\beta}_j x_{ij})$$

$$\Rightarrow \quad V_\theta(\tilde{z}') = V_\theta(\tilde{z}) + V_\theta(\sum_{i=1}^{m} \tilde{\alpha}_i \sum_{j=1}^{m} x_{ij}) + V_\theta(\sum_{j=1}^{m} \tilde{\beta}_j \sum_{i=1}^{m} x_{ij})$$

$$\Rightarrow \quad V_\theta(\tilde{z}') - V_\theta(\tilde{z}) = V_\theta(\sum_{i=1}^{m} \tilde{\alpha}_i) + V_\theta(\sum_{j=1}^{m} \tilde{\beta}_j) \quad [\text{as } \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{m} x_{ij} = 1]$$

$$\Rightarrow \quad V_\theta(\tilde{z}' - \tilde{z}) = \sum_{i=1}^{m} V_\theta(\tilde{\alpha}_i) + \sum_{j=1}^{m} V_\theta(\tilde{\beta}_j)$$

$$\Rightarrow \quad V_\theta(\tilde{z}' - \tilde{z}) = \kappa + \zeta \quad [\text{for } \sum_{i=1}^{m} V_\theta(\tilde{\alpha}_i) = \kappa, \sum_{j=1}^{m} V_\theta(\tilde{\beta}_j) = \zeta]$$

Thus the two objective functions $\tilde{z}'$ and $\tilde{z}$ differ by a constant not involving any decision variable x_{ij} and so the original problem as well as the new problem both attain same optimal solution. □

4.1. Numerical Example

Here, assignment problems with the costs as both SVTN-numbers and SVTrN-numbers have been solved by use of proposed method. For simplicity, we define the θ- weighted value function for $n = 1$ in rest of this paper.

Example 4.1. *A car owner wishes to run his three buses* (B_1, B_2, B_3) *in three different routes* (R_1, R_2, R_3)*. The maintenance cost of each bus per kilometer in three different routes is given by the following cost matrix* $[\tilde{a}_{ij}]$ *whose elements are SVTN-numbers. Allot the route for each bus so that the maintenance cost in aggregate becomes minimum.*

$$[\tilde{a}_{ij}] = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \end{array} \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{pmatrix} \end{array}$$

where

$$\tilde{a}_{11} = \langle[10,13,3,6],[10,13,8,5],[10,13,2,9]\rangle,\ \tilde{a}_{12} = \langle[8,11,4,6],[8,11,6,9],[8,11,5,10]\rangle,$$
$$\tilde{a}_{13} = \langle[5,9,3,5],[5,9,2,9],[5,9,1,12]\rangle,\quad \tilde{a}_{21} = \langle[4,7,3,9],[4,7,1,6],[4,7,2,8]\rangle,$$
$$\tilde{a}_{22} = \langle[7,12,5,12],[7,12,4,8],[7,12,3,10]\rangle,\ \tilde{a}_{23} = \langle[9,10,2,7],[9,10,6,12],[9,10,4,5]\rangle,$$
$$\tilde{a}_{31} = \langle[3,8,1,4],[3,8,2,7],[3,8,2,2]\rangle,\quad \tilde{a}_{32} = \langle[6,7,4,9],[6,7,5,5],[6,7,2,10]\rangle,$$
$$\tilde{a}_{33} = \langle[12,18,10,2],[12,18,8,7],[12,18,5,9]\rangle.$$

The problem can be put in the following form :

$$\text{Min } \tilde{z} =_{\Re_\theta} \tilde{a}_{11}x_{11} + \tilde{a}_{12}x_{12} + \tilde{a}_{13}x_{13} + \tilde{a}_{21}x_{21} + \tilde{a}_{22}x_{22} + \tilde{a}_{23}x_{23} + \tilde{a}_{31}x_{31} + \tilde{a}_{32}x_{32} + \tilde{a}_{33}x_{33}$$
$$\text{such that}\quad x_{11}+x_{12}+x_{13}=1,\ x_{11}+x_{21}+x_{31}=1,$$
$$x_{21}+x_{22}+x_{23}=1,\ x_{12}+x_{22}+x_{32}=1,$$
$$x_{31}+x_{32}+x_{33}=1,\ x_{13}+x_{23}+x_{33}=1;\ \text{with } x_{ij}\in\{0,1\}.$$

The θ - weighted value for the SVTN-numbers are calculated as :

$$V_\theta(\tilde{a}_{11}) = \frac{1}{6}(142-70\theta),\ V_\theta(\tilde{a}_{12}) = \frac{1}{6}(122-63\theta),\ V_\theta(\tilde{a}_{13}) = \frac{1}{6}(102-58\theta),$$
$$V_\theta(\tilde{a}_{21}) = \frac{1}{6}(77-38\theta),\ V_\theta(\tilde{a}_{22}) = \frac{1}{6}(125-61\theta),\ V_\theta(\tilde{a}_{23}) = \frac{1}{6}(121-59\theta),$$
$$V_\theta(\tilde{a}_{31}) = \frac{1}{6}(71-35\theta),\ V_\theta(\tilde{a}_{32}) = \frac{1}{6}(86-42\theta),\ V_\theta(\tilde{a}_{33}) = \frac{1}{6}(183-101\theta).$$

Assuming $\theta = 0.8$, we get the initial assignment Table 2 as follows (each entry taking two decimal places): Now applying the computational procedure as in crisp environment, we find the allocation of busses in different

Table 2. Initial assignment table.

14.33	11.93	9.27
7.77	12.70	12.30
7.17	8.73	17.03

route with the optimal costs attained as follows :

$$B_1 \longrightarrow R_3,\ B_2 \longrightarrow R_1,\ B_3 \longrightarrow R_2.$$

The optimal solutions are :

$$x_{13} = x_{21} = x_{32} = 1,\quad x_{11} = x_{12} = x_{22} = x_{23} = x_{31} = x_{33} = 0.$$

and Min $\tilde{z} =_{\Re_\theta} \tilde{a}_{13} + \tilde{a}_{21} + \tilde{a}_{32}$ which becomes Rs. 25.77 using θ-weighted value function for $\theta = 0.8$.

Remark 4.1. *Depending on θ chosen, the number of iteration in computational procedure to reach at optimality stage may vary only but the optimal solution will remain unchange. However, θ plays an important role to produce the aggregate optimal value of an assignment problem in neutrosophic environment. Since the total maintenance cost of a bus depends on so many factors, we assume θ as the degree of smoothness of road condition in the present problem. Following Table 3 shows the variation of optimal value with respect to different θ in the Example 4.1. Here $V_\theta(\tilde{z}) = \frac{1}{6}(265-138\theta)$.*

Table 3. Table for variation of optimal value.

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$V_\theta(\tilde{z})$	44.17	41.87	39.57	37.27	34.97	32.67	30.37	28.07	25.77	23.47	21.17

Example 4.2. *Four machines (M_1, M_2, M_3, M_4) are available to perform four jobs (J_1, J_2, J_3, J_4) in a company. The machines M_1 and M_3 can not perform the jobs J_3 and J_1 respectively. The adjacent matrix provides the approx required time (in minutes) to perform the jobs by different machines (given in SVTN-numbers and SVTrN-numbers). How would the jobs be allotted to minimize the total time. Provided that one machine will perform one job only.*

$$[\tilde{a}_{ij}] = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & - & \tilde{a}_{14} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{a}_{24} \\ - & \tilde{a}_{32} & \tilde{a}_{33} & \tilde{a}_{34} \\ \tilde{a}_{41} & \tilde{a}_{42} & \tilde{a}_{43} & \tilde{a}_{44} \end{pmatrix} \end{array}$$

where

$$\begin{aligned}
&\tilde{a}_{11} = \langle[5,8,4,3],[5,8,1,9],[5,8,2,6]\rangle, && \tilde{a}_{12} = \langle[10,8,2],[10,2,8],[10,5,13]\rangle, \\
&\tilde{a}_{14} = \langle[7,11,2,5],[7,11,5,9],[7,11,1,7]\rangle, && \tilde{a}_{21} = \langle[15,8,9],[15,4,11],[15,6,10]\rangle, \\
&\tilde{a}_{22} = \langle[3,7,2,4],[3,7,2,6],[3,7,1,10]\rangle, && \tilde{a}_{23} = \langle[5,8,4,10],[5,8,3,12],[5,8,2,9]\rangle, \\
&\tilde{a}_{24} = \langle[6,5,8],[6,2,9],[6,3,5]\rangle, && \tilde{a}_{32} = \langle[10,14,8,7],[10,14,6,10],[10,14,3,4]\rangle, \\
&\tilde{a}_{33} = \langle[9,2,3],[9,1,5],[9,3,8]\rangle, && \tilde{a}_{34} = \langle[8,12,6,5],[8,12,4,10],[8,12,5,11]\rangle, \\
&\tilde{a}_{41} = \langle[8,4,8],[8,2,3],[8,5,10]\rangle, && \tilde{a}_{42} = \langle[9,13,2,5],[9,13,7,8],[9,13,5,9]\rangle, \\
&\tilde{a}_{43} = \langle[7,11,6,5],[7,11,4,3],[7,11,2,8]\rangle, && \tilde{a}_{44} = \langle[12,10,4],[12,7,6],[12,3,3]\rangle.
\end{aligned}$$

Since the machines M_1 and M_3 are unable to perform the jobs J_3 and J_1 respectively, so we can assume two SVN-numbers ($\tilde{a}_{13}$ and $\tilde{a}_{31}$) whose θ-weighted value functions provide very large time in the cells (1,3) and (3,1) respectively.

$$\tilde{a}_{13} = \langle[16,30,2,9],[16,30,8,15],[16,30,7,11]\rangle,\ \tilde{a}_{31} = \langle[18,22,6,13],[18,22,3,17],[18,22,9,28]\rangle.$$

Then the problem can be put in the following form :

$$\begin{aligned}
\text{Min } \tilde{z} \quad =_{\Re_\theta} \quad & \tilde{a}_{11}x_{11} + \tilde{a}_{12}x_{12} + \tilde{a}_{13}x_{13} + \tilde{a}_{14}x_{14} + \tilde{a}_{21}x_{21} + \tilde{a}_{22}x_{22} + \tilde{a}_{23}x_{23} + \tilde{a}_{24}x_{24} \\
& \tilde{a}_{31}x_{31} + \tilde{a}_{32}x_{32} + \tilde{a}_{33}x_{33} + \tilde{a}_{34}x_{34} + \tilde{a}_{41}x_{41} + \tilde{a}_{42}x_{42} + \tilde{a}_{43}x_{43} + \tilde{a}_{44}x_{44} \\
\text{such that} \qquad & x_{11} + x_{12} + x_{13} + x_{14} = 1,\ \ x_{11} + x_{21} + x_{31} + x_{41} = 1, \\
& x_{21} + x_{22} + x_{23} + x_{24} = 1,\ \ x_{12} + x_{22} + x_{32} + x_{42} = 1, \\
& x_{31} + x_{32} + x_{33} + x_{34} = 1,\ \ x_{13} + x_{23} + x_{33} + x_{43} = 1, \\
& x_{41} + x_{42} + x_{43} + x_{44} = 1,\ \ x_{14} + x_{24} + x_{34} + x_{44} = 1; \quad \text{with } x_{ij} \in \{0,1\}.
\end{aligned}$$

The θ - weighted value for the SVTN-numbers are calculated as :

$$V_\theta(\tilde{a}_{11}) = \frac{1}{6}(90-52\theta),\ V_\theta(\tilde{a}_{12}) = \frac{1}{6}(134-80\theta),\ V_\theta(\tilde{a}_{13}) = \frac{1}{6}(287-142\theta),\ V_\theta(\tilde{a}_{14}) = \frac{1}{6}(118-61\theta),$$
$$V_\theta(\tilde{a}_{21}) = \frac{1}{6}(191-100\theta),\ V_\theta(\tilde{a}_{22}) = \frac{1}{6}(73-41\theta),\ V_\theta(\tilde{a}_{23}) = \frac{1}{6}(94-49\theta),\ V_\theta(\tilde{a}_{24}) = \frac{1}{6}(81-42\theta),$$
$$V_\theta(\tilde{a}_{31}) = \frac{1}{6}(273-146\theta),\ V_\theta(\tilde{a}_{32}) = \frac{1}{6}(149-78\theta),\ V_\theta(\tilde{a}_{33}) = \frac{1}{6}(117-62\theta),\ V_\theta(\tilde{a}_{34}) = \frac{1}{6}(132-73\theta),$$
$$V_\theta(\tilde{a}_{41}) = \frac{1}{6}(102-50\theta),\ V_\theta(\tilde{a}_{42}) = \frac{1}{6}(137-68\theta),\ V_\theta(\tilde{a}_{43}) = \frac{1}{6}(113-60\theta),\ V_\theta(\tilde{a}_{44}) = \frac{1}{6}(143-77\theta).$$

Taking $\theta = 0.4$, we get the initial assignment Table 4 as follows (each entry taking two decimal places): The

Table 4. Initial assignment table.

11.53	17	38.37	15.6
25.17	9.43	12.40	10.70
35.77	19.63	15.37	17.13
13.67	18.30	14.83	18.70

computational procedure in crisp environment of Table 4 gives the final iteration as in Table 5. Thus we get

Table 5. Optimal assignment table.

0	3.82	25.68	1.15
17.39	0	3.46	0
21.56	3.77	0	0
0	2.98	0	2.11

the allotment of jobs to the machines with a total minimum time taken as follows :

$$M_1 \longrightarrow J_1,\ \ M_2 \longrightarrow J_2,\ \ M_3 \longrightarrow J_4,\ \ M_4 \longrightarrow J_3.$$

The optimal solutions are :

$x_{11} = x_{22} = x_{34} = x_{43} = 1, \quad x_{12} = x_{13} = x_{14} = x_{21} = x_{23} = x_{24} = x_{31} = x_{32} = x_{33} = x_{41} = x_{42} = x_{44} = 0.$

and Min $\tilde{z} =_{\Re_\theta} \tilde{a}_{11} + \tilde{a}_{22} + \tilde{a}_{34} + \tilde{a}_{43}$ which becomes 52.93 minutes using θ-weighted value function for $\theta = 0.4$.

Remark 4.2. *(Sensitivity analysis in post optimization)*
The variation of optimal time (taking two decimal places) with respect to different θ given in Table 6 for the Example 4.2 Here $V_\theta(\tilde{z}) = \frac{1}{6}(408-226\theta)$ where θ denotes the associate infrastructure facility provided by company, we claim.

Table 6. Table for variation of optimal value

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$V_\theta(\tilde{z})$	68	64.23	60.47	56.7	52.93	49.17	45.4	41.63	37.87	34.1	30.33

5. Conclusion

In this paper, an SVN-number is presented in a new direction with the study of its characteristics. A linear ranking function is then defined to compare two or more such numbers. In continuation, a model of assignment problem is developed in neutrosophic environment where each entry of cost matrix is an SVN-number. The proposed concept is illustrated by two practical problems. The problems are stated and solved numerically. In post optimization period, the sensitivity analysis of these problems also have been performed with respect to different values of parameter. The fact has a nice significance in any kind of system solvable by the proposed approach. The notion of ranking function adopted here will bring a new arena in research and development of linear programming problem and its associated field, we expect.

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