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Contra $n\mathcal{I}_{*\mu}$-continuity

S. Ganesan[1*], P. Hema[2] , S. Jeyashri[3] and C. Alexander[4]
[1,4]Assistant Professor, PG & Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.
(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)
[2]Research Scholar, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal-624102, Tamil Nadu, India.
[3]Assistant Professor, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal-624102, Tamil Nadu, India

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Abstract: In this paper, $n\mathcal{I}_{*\mu}$-closed sets and $n\mathcal{I}_{*\mu}$-open sets are used to define and investigate a new class of maps called contra $n\mathcal{I}_{*\mu}$-continuous maps in nano ideal topological spaces. We discuss the relationship with some other related maps.

Key words: $n\mathcal{I}_{*\mu}$-closed sets, $n\mathcal{I}_{*\mu}$-continuity, $n\mathcal{I}_{*\mu}$-irresolute map, contra n$^*\mu$-continuity and contra $n\mathcal{I}_{*\mu}$-continuity

1. Introduction

In 1960, Jankovic and Hamlett [9], [10] have considered the local function in ideal topological space any they have obtained a new topology. In 2016, Parimala et. al.[15] introduce a similar type with the local function in nano topological spaces. Before starting the discussion we shall consider the following concepts.

Let (O, $\mathcal{N}$) be a nano topological space, where $N = \tau_R(X)$.

A nano topological space (O, $\mathcal{N}$) with an ideal $\mathcal{I}$ on O is called a nano ideal topological space and is denoted by (O, $\mathcal{N}$, $\mathcal{I}$).

Let (O, $\mathcal{N}$) be a nano topological space and $G_n(o) = \{G_n \mid k \in G_n, G_n \in \mathcal{N}\}$ be the family of nano open sets which contain o.

Let (O, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on O, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n : \wp(O) \to \wp(O)$ ($\wp(O)$ is the set of all subsets of O) [15, 16]. For a subset A $\subseteq$ O, $A^*_n(\mathcal{I}, \mathcal{N}) = \{o \in O : G_n \cap A \notin \mathcal{I}$, for every $G_n \in G_n(o)\}$, where $G_n(o) = \{G_n \mid o \in G_n, G_n \in \mathcal{N}\}$ is called the nano local function (briefly n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for $A^*_n(\mathcal{I}, \mathcal{N})$.

Parimala et al [16] introduced the concept of nano ideal generalized closed sets in nano ideal topological spaces and investigated some of its basic properties. Ganesan [7] introduced and studied $n\mathcal{I}_{*\mu}$-continuous map and $n\mathcal{I}_{*\mu}$-irresolute map in nano ideal topological spaces. Recently, Ganesan [6] introduced and studied contra $n\mathcal{I}_g$-continuity in nano ideal topological spaces. In this paper, we introduce the notation of contra $n\mathcal{I}_{*\mu}$-continuity in nano ideal topological spaces and discuss their properties and give various characterizations.

*Correspondence: sgsgsgsgsg77@gmail.com

2. Preliminaries

Definition 2.1. [15, 16] A subset A of a nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be n$\star$-closed if A_n^* $\subseteq$ A.

Lemma 2.1. *[15, 16] Let (O, $\mathcal{N}$, $\mathcal{I}$) be a nano topological space with an ideal $\mathcal{I}$ and A$\subseteq A_n^*$, then A_n^*=n-cl(A_n^*)=n-cl(A)*

Definition 2.2. [14] A subset M of a space (U, $\tau_R(X)$) is said to be nano pre-closed set if ncl(nint(M)) $\subseteq$ M. The complement of a nano pre-closed set is said to be nano pre-open.

Definition 2.3. A subset M of a space (U, $\tau_R(X)$) is said to be n$^*\mu$-closed [5] if nclo(A) $\subseteq$ V whenever A $\subseteq$ V and V is n*gs-open in (U, $\tau_R(X)$). The complement of a n$^*\mu$-closed set is said to be n$^*\mu$-open set.

Definition 2.4. A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be

1. nano-$\mathcal{I}$-generalized closed (briefly, $n\mathcal{I}_g$-closed) [15, 16] if A_n^* $\subseteq$ V whenever A$\subseteq$ V and V is n-open.
2. $n\mathcal{I}_{*\mu}$-closed [4] if A_n^* $\subseteq$ V whenever A$\subseteq$ V and V is n*gs-open.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 2.5. [13] A nano topological space (U, $\tau_R(X)$) is said to nano locally indiscrete if every n-open set is n-closed.

Definition 2.6. [2] A nano topological space (U, $\tau_R(X)$) is said to be nano-regular space, if for each nano closed set F and each point x $\notin$ F , there exists disjoint nano open sets G and H such that x $\in$ G and F $\subset$ H

Definition 2.7. [12] A nano topological space (U, $\tau_R(X)$) is said to be nano-connected if (U, $\tau_R(X)$) cannot be expressed as a disjoint union of two non-empty nano-open sets. A subset of (U, $\tau_R(X)$) is nano-connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano-connected

Definition 2.8. [8] A map f: (O, $\mathcal{N}$) $\rightarrow$ (P, $\mathcal{N}'$) is said to be contra n$^*\mu$-continuous if f^{-1}(V) is a n$^*\mu$-closed set of (O, $\mathcal{N}$) for every n-open set V of (P, $\mathcal{N}'$).

Definition 2.9. A map f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) is said to be n$\star$-continuous [11] (resp. $n\mathcal{I}_{*\mu}$-continuous [4]) if f^{-1}(A) is n$\star$-closed (resp. $n\mathcal{I}_{*\mu}$-closed) in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$).

Definition 2.10. [7] A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called $n\mathcal{I}_{*\mu}$-irresolute if f^{-1}(V) is a $n\mathcal{I}_{*\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every $n\mathcal{I}_{*\mu}$-closed set V of (P, $\mathcal{N}'$, $\mathcal{J}$).

Theorem 2.1. 1. *Every n-closed is n$\star$-closed set but not conversely [1, 3].*

2. *Every n$\star$-closed set is $n\mathcal{I}_{*\mu}$-closed but not conversely [4].*
3. *every $n\mathcal{I}_{*\mu}$-closed set is $n\mathcal{I}_g$-closed but not conversely [4].*

Definition 2.11. [6] A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) is said to be contra n$\star$-continuous (resp. contra $n\mathcal{I}_g$-continuous) if f^{-1}(G) is a n$\star$-closed (resp. $n\mathcal{I}_g$-closed) in (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set G of (P, $\mathcal{N}'$).

3. Contra $n\mathcal{I}_{^*\mu}$-continuity

Let (O, $\mathcal{N}$, $\mathcal{I}$) (or O) represent nano ideal topological spaces on which no separation axioms are assumed unless otherwise mentioned.

Let (P, $\mathcal{N}'$) (or P) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned.

We introduce the following definition

Definition 3.1. A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) is said to be contra $n\mathcal{I}_{^*\mu}$-continuous if f^{-1}(G) is a $n\mathcal{I}_{^*\mu}$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set G of (P, $\mathcal{N}'$).

Proposition 3.1. *Every contra n$\star$-continuous map is contra $n\mathcal{I}_{^*\mu}$-continuous.*

Proof. Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be a contra n$\star$-continuous map and let G be any n-open set in (P, $\mathcal{N}'$). Then, f^{-1}(G) is n$\star$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Since every n$\star$-closed set is $n\mathcal{I}_{^*\mu}$-closed, f^{-1}(G) is $n\mathcal{I}_{^*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Therefore f is contra $n\mathcal{I}_{^*\mu}$-continuous. □

Example 3.1. *Let O = {8, 9, 10}, with O/ R= {{10}, {8, 9}, {9, 8}} and X= {8, 9}. Then the Nano topology $\mathcal{N}$ = {ϕ, {8, 9}, O} and $\mathcal{I}$={$\emptyset$, {8}}. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and Y = {8, 9}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P}. Then n$\star$-closed sets are ϕ, O, {8}, {10}, {8, 10} and $n\mathcal{I}_{^*\mu}$-closed sets are ϕ, O, {8}, {10}, {8, 10}, {9, 10}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be the identity map. Then f is contra $n\mathcal{I}_{^*\mu}$-continuous but not contra n$\star$-continuous, since f^{-1}({9, 10})= {9, 10} is not n$\star$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Proposition 3.2. *Every contra $n\mathcal{I}_{^*\mu}$-continuous map is contra $n\mathcal{I}_g$-continuous.*

Proof. Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be a contra $n\mathcal{I}_{^*\mu}$-continuous map and let G be any n-open set in (P, $\mathcal{N}'$). Then, f^{-1}(G) is $n\mathcal{I}_{^*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Since every $n\mathcal{I}_{^*\mu}$-closed set is $n\mathcal{I}_g$-closed, f^{-1}(G) is $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Therefore f is contra $n\mathcal{I}_g$-continuous. □

Example 3.2. *Let O = {8, 9, 10}, with O/ R= {{8}, {9, 10}} and X= {8}. Then the Nano topology $\mathcal{N}$ = {ϕ, {8}, O} and $\mathcal{I}$={$\emptyset$, {8}}. Let P = {8, 9, 10}, with P/ R= {{9}, {8, 10}, {10, 8}} and Y = {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8, 10}, P}. Then $n\mathcal{I}_{^*\mu}$-closed sets are ϕ, O, {8}, {9, 10} and $n\mathcal{I}_g$-closed sets are ϕ, O, {8}, {9}, {10}, {8, 9}, {8, 10}, {9, 10}. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be the identity map. Then f is contra $n\mathcal{I}_g$-continuous but not contra $n\mathcal{I}_{^*\mu}$-continuous, since f^{-1}({8, 10})= {8, 10} is not $n\mathcal{I}_{^*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Proposition 3.3. *Every contra n$^*\mu$-continuous map is contra $n\mathcal{I}_{^*\mu}$-continuous.*

Proof. Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be a contra n$^*\mu$-continuous map and let G be any n-open set in (P, $\mathcal{N}'$). Then, f^{-1}(G) is n$^*\mu$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Since every n$^*\mu$-closed set is $n\mathcal{I}_{^*\mu}$-closed, f^{-1}(G) is $n\mathcal{I}_{^*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Therefore f is contra $n\mathcal{I}_{^*\mu}$-continuous. □

Example 3.3. *Let (O , $\mathcal{N}$, $\mathcal{I}$) and f be defined as an Example 3.2. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and Y = {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P}. Then n$^*\mu$-closed sets are ϕ, O, {9, 10}. Then f is contra $n\mathcal{I}_g$-continuous but not contra n$^*\mu$-continuous, since f^{-1}({8})= {8} is not n$^*\mu$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Remark 3.1. *The following example shows that $n^*\mu$-continuity and contra $n\mathcal{I}_{*\mu}$-continuity are independent.*

Example 3.4. *Let (O , $\mathcal{N}$, $\mathcal{I}$) and f be defined as an Example 3.1. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}, {10, 9}} and Y = {9, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {9, 10}, P}. Then $n^*\mu$-closed sets are ϕ, O, {10}, {8, 10}, {9, 10}. Then f is contra $n\mathcal{I}_{*\mu}$-continuous but not $n^*\mu$-continuous, since f^{-1}({8})= {8} is not $n^*\mu$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Example 3.5. *Let (O , $\mathcal{N}$, $\mathcal{I}$) and f be defined as an Example 3.4. Let P = {8, 9, 10}, with P/ R= {{9}, {8, 10}} and Y = {9}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {9}, P}. Then f is $n^*\mu$-continuous but not contra $n\mathcal{I}_{*\mu}$-continuous, since f^{-1}({9})= {9} is not $n\mathcal{I}_{*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Remark 3.2. *The composition of two contra $n\mathcal{I}_{*\mu}$-continuous maps need not be contra $n\mathcal{I}_{*\mu}$-continuous and this is shown from the following example.*

Example 3.6. *Let O = {8, 9, 10}, with O/ R= {{8}, {9, 10}} and X= {8}. Then the Nano topology $\mathcal{N}$ = {ϕ, {8}, O} and $\mathcal{I}$ = {$\emptyset$, {8}}. Then $n\mathcal{I}_{*\mu}$-closed sets are ϕ, O, {8}, {9, 10}. Let P = {8, 9, 10}, with P/ R= {{8}, {9, 10}} and Y = {8, 10}. Then the Nano topology $\mathcal{N}'$ = {ϕ, {8}, {9, 10}, P} and $\mathcal{J}$ = {$\emptyset$}. Then $n\mathcal{I}_{*\mu}$-closed sets are ϕ, P, {8}, {9}, {10}, {8, 9}, {8, 10}, {9, 10}. Let Q = {8, 9, 10} with Q/ R= {{10}, {8, 9}, {9, 8}} and Z = {8, 9}. Then the Nano topology $\mathcal{N}'_*$ = $\{\phi, \{8,9\}, Q\}$. Define $f : (O,\mathcal{N},\mathcal{I}) \to$ (P, $\mathcal{N}'$, $\mathcal{J}$) and $g : (P,\mathcal{N}',\mathcal{J}) \to$ (Q, $\mathcal{N}'_*$) be the identity maps. Clearly f and g are contra $n\mathcal{I}_{*\mu}$-continuous but their $g \circ f$: (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (Q,$\mathcal{N}'_*$) is not contra $n\mathcal{I}_{*\mu}$-continuous, because V = {8, 9} is n-open in (Q, $\mathcal{N}'_*$) but ($g \circ f$ $^{-1}$({8, 9}) = f^{-1}(g^{-1}({8, 9})) = f^{-1}({8, 9}) = {8, 9}, which is not $n\mathcal{I}_{*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).*

Theorem 3.1. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (P, $\mathcal{N}'$) be a map. Then the following conditions are equivalent*

1. *f is contra $n\mathcal{I}_{*\mu}$-continuous.*
2. *The inverse image of each n-open set in P is $n\mathcal{I}_{*\mu}$-closed in O.*
3. *The inverse image of each n-closed set in P is $n\mathcal{I}_{*\mu}$-open in O.*
4. *For each point o in O and each n-closed set G in P with f(o) $\in$ G, there is an $n\mathcal{I}_{*\mu}$-open set U in O containing o such that f(U) $\subset$ G.*

Proof. (1) $\Rightarrow$ (2). Let G be n-open in P. Then P $-$ G is n-closed in P. By definition of contra $n\mathcal{I}_{*\mu}$-continuous, f^{-1}(P $-$ G) is $n\mathcal{I}_{*\mu}$-open in O. But f^{-1}(P $-$ G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_{*\mu}$-closed in O.
(2) $\Rightarrow$ (3) Let G be any n-closed set in P. Then P$-$G is n-open set in P. By the assumption of (2), f^{-1}(P $-$ G) is $n\mathcal{I}_{*\mu}$-closed in O. But f^{-1}(P $-$ G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_{*\mu}$-open in O.
(3) $\Rightarrow$ (4). Let o $\in$ O and G be any n-closed set in P with f(o) $\in$ G. By (3), f^{-1}(G) is $n\mathcal{I}_{*\mu}$-open in O. Set U= f^{-1}(G). Then there is an $n\mathcal{I}_{*\mu}$-open set U in O containing o such that f(U) $\subset$ G.
(4) $\Rightarrow$ (1). Let o $\in$ O and G be any n-closed set in P with f(o) $\in$ G. Then P $-$ G is n-open in P with f(o) $\in$ G. By (4), there is an $n\mathcal{I}_{*\mu}$-open set U in O containing o such that f(U) $\subset$ G. This implies U= f^{-1}(G) . Therefore, O $-$ U = O $-$ f^{-1}(G) = f^{-1}(P$-$G) which is $n\mathcal{I}_{*\mu}$-closed in O. □

Theorem 3.2. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\to$ (P, $\mathcal{N}'$) and g: (P, $\mathcal{N}'$, $\mathcal{J}$) $\to$ (Q, $\mathcal{N}'_*$). Then the following properties hold:*

1. *If f is contra $n\mathcal{I}_{*\mu}$-continuous and g is $n\star$-continuous then $g \circ f$ is contra $n\mathcal{I}_{*\mu}$-continuous.*
2. *If f is contra $n\mathcal{I}_{*\mu}$-continuous and g is contra $n\star$-continuous then $g \circ f$ is $n\mathcal{I}_{*\mu}$-continuous.*
3. *If f is $n\mathcal{I}_{*\mu}$-continuous and g is contra $n\star$-continuous then $g \circ f$ is contra $n\mathcal{I}_{*\mu}$-continuous.*

Proof. (1) Let G be n-closed set in Q. Since g is n$\star$-continuous, g^{-1}(G) is n-closed in P. Since f is contra $n\mathcal{I}_{*\mu}$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_{*\mu}$-open in O. Therefore g ∘ f is contra $n\mathcal{I}_{*\mu}$-continuous.
(2) Let G be any n-closed set in Q. Since g is contra n$\star$-continuous, g^{-1}(G) is n-open in P. Since f is contra $n\mathcal{I}_{*\mu}$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_{*\mu}$-closed in O. Therefore g ∘ f is $n\mathcal{I}_{*\mu}$-continuous.
(3) Let G be any n-closed set in Q. Since g is contra n$\star$-continuous, g^{-1}(G) is n-open in P. Since f is $n\mathcal{I}_{*\mu}$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_{*\mu}$-open in O. Therefore g ∘ f is contra $n\mathcal{I}_{*\mu}$-continuous. □

Theorem 3.3. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_{*\mu}$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$) is contra $n\star$-continuous map, then $g \circ f$: (O, $\mathcal{N}$, $\mathcal{I}$) → (Q, $\mathcal{N}'_*$) is contra $n\mathcal{I}_{*\mu}$-continuous map.*

Proof. Since g is contra n$\star$-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q, $\mathcal{N}'_*$), for any n-open set in q as a subset of Q, we get, $g^{-1}(q) = Gisan - closedsetin(P,\mathcal{N}',\mathcal{J})$. By Theorem 2.1 (1) and (2), it implies that g^{-1}(q) = G is $n\mathcal{I}_{*\mu}$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_{*\mu}$-irresolute map. We get (g ∘ f)$^{-1}$(q)= f^{-1}(g^{-1}(q)) = f^{-1}(G) = S and S is a $n\mathcal{I}_{*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_{*\mu}$-continuous map. □

Theorem 3.4. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_{*\mu}$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) → (Q,N'_*) is contra $n\mathcal{I}_{*\mu}$-continuous map, then $g \circ f : (O,\mathcal{N},\mathcal{I} \to (Q,\mathcal{N}'_*)$ is contra $n\mathcal{I}_{*\mu}$-continuous map.*

Proof. Since g is contra $n\mathcal{I}_{*\mu}$-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) → $(Q,\mathcal{N}'_*)$, for any n-open set in q as a subset of Q, we get, $g^{-1}(q) = G$ is a $n\mathcal{I}_{*\mu}$-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_{*\mu}$-irresolute map. We get $(g \circ f)^{-1}(q) = f^{-1}(g^{-1}(q)) = f^{-1}(G) = S$ and S is a $n\mathcal{I}_{*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_{*\mu}$-continuous map. □

Theorem 3.5. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) be a map and g : (O, $\mathcal{N}$, $\mathcal{I}$) → ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$)) the graph map of f, defined by g(o) = (o, f(o)) for every o ∈ O. If g is contra $n\mathcal{I}_{*\mu}$-continuous, then f is contra $n\mathcal{I}_{*\mu}$-continuous.*

Proof. Let G be an n-open set in (P, $\mathcal{N}'$). Then ((O, $\mathcal{N}$, $\mathcal{I}$) × G) is an n-open set in ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$). It follows from Theorem 3.1, that f^{-1}(G) = g^{-1}((O, $\mathcal{N}$, $\mathcal{I}$) × G) is $n\mathcal{I}_{*\mu}$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Thus, f is contra $n\mathcal{I}_{*\mu}$-continuous. □

Theorem 3.6. *If a map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$) is contra $n\mathcal{I}_{*\mu}$-continuous and P is nano regular, then f is $n\mathcal{I}_{*\mu}$-continuous.*

Proof. Let o be an arbitrary point of O and G be any n-open set of P containiing f(o). Since P is nano regular, there exists an n-open set W in P containing f(o) such that A^*_n(W) ⊂ G. Since f is contra $n\mathcal{I}_{*\mu}$-continuous, by Theorem 3.1, there exists an $n\mathcal{I}_{*\mu}$-open set U containing o such that f(U) ⊂ A^*_n(W). Thus f(U) ⊂ A^*_n(W) ⊂ G. Hence f is $n\mathcal{I}_{*\mu}$-continuous. □

Definition 3.2. A space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be an $n\mathcal{I}_{*\mu}$-space if every $n\mathcal{I}_{*\mu}$-open set is n-open in (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 3.7. *A map f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) is contra $n\mathcal{I}_{*\mu}$-continuous and O is $n\mathcal{I}_{*\mu}$-space, then f is contra n$\star$-continuous.*

Proof. Let G be n-closed set in P. Since f is contra $n\mathcal{I}_{*\mu}$-continuous, f^{-1}(G) is $n\mathcal{I}_{*\mu}$-open in O. Since O is an $n\mathcal{I}_{*\mu}$-space, f^{-1}(G) is n-open in O. Therefore f is contra n$\star$-continuous. □

Definition 3.3. An nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be $n\mathcal{I}_{*\mu}$-connected if (O, $\mathcal{N}$, $\mathcal{I}$) cannot be expressed as the union of two disjoint non empty $n\mathcal{I}_{*\mu}$-open subsets of (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 3.8. *A contra $n\mathcal{I}_{*\mu}$-continuous image of a $n\mathcal{I}_{*\mu}$-connected space is nano connected.*

Proof. Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be a contra $n\mathcal{I}_{*\mu}$-continuous map of an $n\mathcal{I}_{*\mu}$-connected space (O, $\mathcal{N}$, $\mathcal{I}$) onto a nano topological space (P, $\mathcal{N}'$). If possible, let P be nano disconnected. Let G and S form a nano disconnection of P. Then G and S are nano clopen and P = G $\cup$ S where G $\cap$ S = ϕ. Since f is contra $n\mathcal{I}_{*\mu}$-continuous, O = f^{-1}(P) = f^{-1}(G $\cup$ S) = f^{-1}(G) $\cup$ f^{-1}(S), where f^{-1}(G) and f^{-1}(S) are non empty $n\mathcal{I}_{*\mu}$-open sets in O. Also f^{-1}(G) $\cap$ f^{-1}(S) = ϕ. Hence O is not $n\mathcal{I}_{*\mu}$-connected. This is a contradiction. Therefore P is nano connected. □

Lemma 3.1. *For an nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$), the following are equivalent.*

1. *O is $n\mathcal{I}_{*\mu}$-connected.*
2. *The only subset of O which are both $n\mathcal{I}_{*\mu}$-open and $n\mathcal{I}_{*\mu}$-closed are the empty set ϕ and O.*

Proof. (1) $\Rightarrow$ (2) Let G be an $n\mathcal{I}_{*\mu}$-open and $n\mathcal{I}_{*\mu}$-closed subset of O. Then O $-$ G is both $n\mathcal{I}_{*\mu}$- open and $n\mathcal{I}_{*\mu}$-closed. Since O is $n\mathcal{I}_{*\mu}$-connected, O can be expressed as union of two disjoint non empty $n\mathcal{I}_{*\mu}$-open sets O and O $-$ G, which implies O $-$ G is empty.
(2) $\Rightarrow$ (1) Suppose O = G $\cup$ S where G and S are disjoint non empty $n\mathcal{I}_{*\mu}$-open subsets of O. Then G is both $n\mathcal{I}_{*\mu}$-open and $n\mathcal{I}_{*\mu}$-closed. By assumption either G = ϕ or O which contradicts the assumption G and S are disjoint non empty $n\mathcal{I}_{*\mu}$-open subsets of O. Therefore O is $n\mathcal{I}_{*\mu}$-connected. □

Definition 3.4. [6] A map f : (O, $\mathcal{N}$) $\rightarrow$ (P, $\mathcal{N}'$) is called nano preclosed if the image of every nano closed subset of O is nano preclosed in P.

Theorem 3.9. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$) be a surjective nano preclosed contra $n\mathcal{I}_{*\mu}$-continuous map. If O is an $n\mathcal{I}_{*\mu}$-space, then P is nano locally indiscrete.*

Proof. Suppose that G is n-open in P. By hypothesis f is contra $n\mathcal{I}_{*\mu}$-continuous and therefore f^{-1}(G) = U is $n\mathcal{I}_{*\mu}$-closed in O. Since O is an $n\mathcal{I}_{*\mu}$-space, U is n-closed in O. Since f is nano preclosed, then G is also nano preclosed in P. Now we have ncl(G) = ncl(nint(G)) $\subset$ G. This means that G is n-closed and hence P is nano locally indiscrete. □

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