



## Decomposition of $n\alpha$ -continuity and $n^*\mu_\alpha$ -continuity

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Received: 19 Jul 2020

Accepted: 03 Aug 2020

Published Online: 20 Aug 2020

**Abstract:** The aim of this paper, we introduce the concepts of  $n^t\eta$ -sets,  $n^{tt}\eta$ -sets,  $n^t\eta$ -continuity,  $n^{tt}\eta$ -continuity & to find decomposition of  $n\alpha$  continuity &  $n^*\mu_\alpha$  continuity repectively in nano topological spaces.

**Key words:**  $n\eta$ -sets,  $n^t\eta$ -set,  $n^{tt}\eta$ -set,  $n\eta$ -continuityt,  $n^t\eta$ -continuity &  $n^{tt}\eta$ -continuity

### 1. Introduction

Jayalakshmi and Janaki [5] introduced and studied the notions of  $n\tau$ -sets,  $nA$ -sets &  $nB$ -sets in nano topological spaces. Recently, Ganesan [4] introduced & studied  $n\alpha B$ -sets,  $n\eta$ -sets,  $n\eta\zeta$ -sets & to find a decomposition of nano continuity. In this paper, we introduce & study the notions of  $n^t\eta$ -sets,  $n^{tt}\eta$ -sets,  $n^t\eta$ -continuity,  $n^{tt}\eta$ -continuity & obtain decomposition of  $n\alpha$  continuity &  $n^*\mu_\alpha$  continuity. Moreover the study of  $n^t\eta$ -sets,  $n^{tt}\eta$ -sets led to some decomposition nano continuity are extensively developed and used in computer science & digital topology.

### 2. Preliminaries

**Definition 2.1.** [7]

If  $(J, \tau_R(P))$  is the nano topological space with respect to  $P$  where  $P \subseteq J$  & if  $M \subseteq J$ , then

- (i) The  $n$ -interior of the set  $M$  is defined as the union of all  $n$ -open subsets contained in  $M$  and it is denoted by  $n\text{inte}(M)$ . That is,  $n\text{inte}(M)$  is the largest  $n$ -open subset of  $M$ .
- (ii) The  $n$ -closure of the set  $M$  is defined as the intersection of all  $n$ -closed sets containing  $M$  and it is denoted by  $n\text{clo}(M)$ . That is,  $n\text{clo}(M)$  is the smallest  $n$ -closed set containing  $M$ .

**Definition 2.2.** [7]

A subset  $M$  of a space  $(J, \tau_R(P))$  is called:

- (i)  $n\alpha$ -closed if  $n\text{clo}(n\text{inte}(n\text{clo}(M))) \subseteq M$ .
- (ii)  $n$ -semi-closed if  $n\text{inte}(n\text{clo}(M)) \subseteq M$ .
- (iii)  $n$ -pre-closed if  $n\text{clo}(n\text{inte}(M)) \subseteq M$ .

(iv)  $n$ -regular-closed if  $\text{ncl}(\text{nint}(M)) = M$ .

The complements of the above mentioned  $n$ -closed are called their respective  $n$ -open.

**Definition 2.3.** A subset  $M$  of a space  $(J, \tau_R(P))$  is called:

- (i) a  $nt$ -set [5] if  $\text{nint}(\text{ncl}(M)) = \text{nint}(M)$ .
- (ii) an  $nA$ -set [5] if  $M = S \cap G$  where  $S$  is  $n$ -open and  $G$  is a  $n$ -regular-closed.
- (iii) a  $nB$ -set [5] if  $M = S \cap G$  where  $S$  is  $n$ -open and  $G$  is a  $nt$ -set.
- (iv) a  $n$ -locally closed set [1] if  $M = S \cap G$  where  $S$  is  $n$ -open and  $G$  is  $n$ -closed.
- (v) an  $n\alpha B$ -set [4] if  $M = S \cap G$  where  $S$  is  $n\alpha$ -open and  $G$  is a  $nt$ -set.
- (vi) an  $n\eta$ -set [4] if  $M = S \cap G$  where  $S$  is  $n$ -open and  $G$  is an  $n\alpha$ -closed.

Collection of  $nt$ -sets (respectively  $nA$ -sets,  $nB$ -sets,  $n$ -locally closed sets,  $n\alpha B$ -set,  $n\eta$ -set) in  $J$  is noted that  $nt(J)$  (respectively  $nA(J)$ ,  $nB(J)$ ,  $nLC(J)$ ,  $n\alpha B(J)$ ,  $n\eta(J)$ ).

**Definition 2.4.** A subset  $M$  of a space  $(J, \tau_R(P))$  is called

- (i) a  $n\hat{g}$ -closed [6] if  $\text{ncl}(M) \subseteq T$  whenever  $M \subseteq T$  and  $T$  is  $n$ -semi-open in  $(J, \tau_R(P))$ . The complement of  $n\hat{g}$ -closed set is called  $n\hat{g}$ -open.
- (ii)  $n^*gs$ -closed [2] if  $\text{nscl}(M) \subseteq T$  whenever  $M \subseteq T$  and  $T$  is  $n\hat{g}$ -open in  $(J, \tau_R(P))$ . The complement of  $n^*gs$ -closed set is called  $n^*gs$ -open.
- (iii)  $n^*\mu_\alpha$ -closed [2] if  $n\alpha\text{cl}(M) \subseteq T$  whenever  $M \subseteq T$  and  $T$  is  $n^*gs$ -open in  $(J, \tau_R(P))$ . The complement of  $n^*\mu_\alpha$ -closed set is called  $n^*\mu_\alpha$ -open.
- (iv)  $n^*\mu_p$ -closed [2] if  $\text{npclo}(M) \subseteq T$  whenever  $M \subseteq T$  and  $T$  is  $n^*gs$ -open in  $(J, \tau_R(P))$ . The complement of  $n^*\mu_p$ -closed set is called  $n^*\mu_p$ -open.

**Proposition 2.1.** (i) Every  $n\alpha$ -open is  $n^*\mu_\alpha$ -open [2].

(ii) Every  $n^*\mu_\alpha$ -open is  $n^*\mu_p$ -open [2].

(iii) Every  $n^*\mu_\alpha$ -continuous is  $n^*\mu_p$ -continuous [3].

**Theorem 2.1.** (i) Every  $n$ -closed is  $Nt$ -set [5].

(ii) Every  $n\alpha$  closed is  $n$ -semi-closed [9].

(iii) Every  $nt$ -set is  $nB$ -set [5].

**Theorem 2.2.** [5]

(i)  $M$  is  $nt$ -set iff it is  $n$ -semi-closed.

(ii) Intersection two  $nt$ -sets is also a  $nt$ -set.

### 3. $n^t\eta$ -sets & $n^{\iota}\eta$ -sets

**Definition 3.1.** A subset M of a space J is called

- (i) an  $n^t\eta$ -set if  $M = S \cap G$  where S is  $n^*$ gs-open and G is  $n\alpha$ -closed.
- (ii) an  $n^{\iota}\eta$ -set if  $M = S \cap G$  where S is  $n^*\mu_\alpha$ -open and G is a nt-set.

Collection of all  $n^t\eta$ -sets (respectively  $n^{\iota}\eta$ -sets) in J will be note that  $n^t\eta(J)$  (respectively  $n^{\iota}\eta(J)$ ).

**Proposition 3.1.** Every  $n\eta$ -set is  $n^t\eta$ -set.

*Proof.* Take E be  $n\eta$ -set. Then  $E = S \cap G$ , where S is n-open and G is  $n\alpha$ -closed. Since every n-open is  $n^*$ gs open, S is  $n^*$ gs open. Hence E is  $n^t\eta$ -set. □

**Example 3.1.** Take  $J = \{1, 2, 3, 4\}$  with  $J/R = \{\{3\}, \{4\}, \{1, 2\}\}$  &  $P = \{2\}$ . The  $n\tau_R(P) = \{\phi, \{1, 2\}, J\}$ . Then  $n^t\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}$  &  $n\eta$ -set are  $\phi, J, \{3\}, \{4\}, \{1, 2\}, \{3, 4\}$ . However, it is clear that  $\{1, 3\}$  is  $n^t\eta$ -set but it is not  $n\eta$ -set.

**Proposition 3.2.** Every  $n\alpha B$ -set is  $n^{\iota}\eta$ -set.

*Proof.* Take E be  $n\alpha B$ -set. Then  $E = S \cap G$ , where S is  $n\alpha$ -open and G is nt-set. Since every  $n\alpha$ -open set is  $n^*\mu_\alpha$ -open, S is  $n^*\mu_\alpha$ -open. Hence E is  $n^{\iota}\eta$ -set. □

**Example 3.2.** Take J &  $n\tau_R(P)$  see Example 3.1. Then  $n^{\iota}\eta$ -set are  $\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}$  &  $n\alpha B$ -set are  $\phi, J, \{3\}, \{4\}, \{1, 2\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}$ . However, it is clear that  $\{1\}$  is  $n^{\iota}\eta$ -set but it is not  $n\alpha B$ -set.

**Proposition 3.3.** Every  $n^*\mu_\alpha$ -open set is  $n^{\iota}\eta$ -set.

*Proof.* Using Definitions 2.4(iii) and 3.1(ii). □

**Example 3.3.** Take J &  $n\tau_R(P)$  see Example 3.2. Then  $n^*\mu_\alpha$ -open are  $\phi, J, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}$ . It is clear that  $\{3, 4\}$  is  $n^{\iota}\eta$ -set but it is not  $n^*\mu_\alpha$ -open.

**Remark 3.1.** (i)  $n^t\eta$ -sets &  $n^*\mu_\alpha$ -closed are independent.

(ii)  $n^{\iota}\eta$ -sets &  $n^*\mu_p$ -closed are independent.

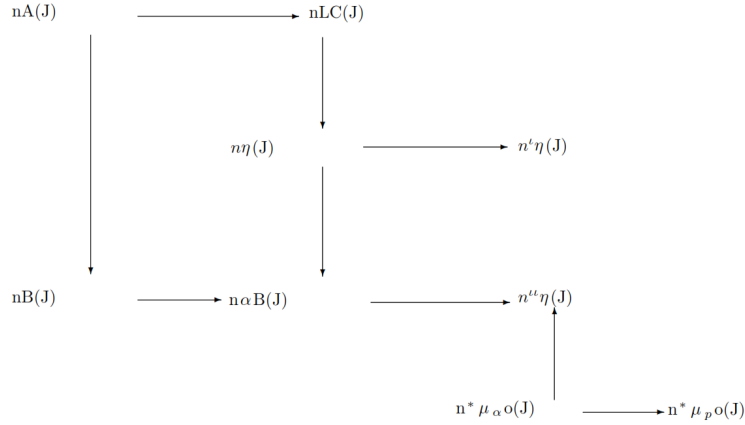
**Example 3.4.** (i) Take J &  $n\tau_R(P)$  see Example 3.1. Then  $n^*\mu_\alpha$ -closed are  $\phi, J, \{3\}, \{4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$ . However, it is clear that  $\{1, 3, 4\}$  is  $n^*\mu_\alpha$ -closed but not  $n^t\eta$ -set & also it is clear that  $\{1, 2, 3\}$  is  $n^t\eta$ -set but not  $n^*\mu_\alpha$ -closed in  $(J, \tau_R(P))$ .

(ii) Let J &  $n\tau_R(P)$  see Example 3.2. Then  $n^*\mu_p$ -closed are  $\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$ . However, it is clear that  $\{1, 3\}$  is  $n^*\mu_p$ -closed but not  $n^{\iota}\eta$ -set & also it is clear that  $\{1, 2, 4\}$  is  $n^{\iota}\eta$ -set but not  $n^*\mu_p$ -closed in  $(J, \tau_R(P))$ .

**Remark 3.2.** We discuss above results see the diagram.

where none of these implications is reversible as shown by [4].

**Theorem 3.1.** For a subset M of a space J, the following conditions are equivalent.



(i)  $M$  is an  $n^t\eta$ -set.

(ii)  $M = S \cap n\alpha clo(M)$  for some  $n^*$ gs-open  $S$ .

*Proof.* (i)  $\rightarrow$  (ii) Since  $M$  is an  $n^t\eta$ -set, then  $M = S \cap G$ , where  $S$  is  $n^*$ gs-open and  $G$  is  $n\alpha$  closed. So,  $M \subseteq S$  and  $M \subseteq G$ . Hence  $n\alpha clo(M) \subseteq n\alpha clo(G)$ . Therefore  $M \subseteq S \cap n\alpha clo(M) \subseteq S \cap n\alpha clo(G) = S \cap G = M$ . Thus,  $M = S \cap n\alpha clo(M)$ .

(ii)  $\rightarrow$  (i) It is obvious because  $n\alpha clo(M)$  is  $n\alpha$ -closed. (Since  $M$  is  $n\alpha$ -closed iff  $M = n\alpha clo(M)$ ). □

**Remark 3.3.** Intersection of two  $n^t\eta$ -sets is an  $n^t\eta$ -set.

**Remark 3.4.** Union of two  $n^t\eta$ -sets need not be an  $n^t\eta$ -set.

**Example 3.5.** Take  $J$  &  $n\tau_R(P)$  see Example 3.1. However, it is clear that  $\{1, 3\}, \{4\}$  are  $n^{\iota}\eta$ -sets in  $(J, \tau_R(P))$  but their union  $\{1, 3, 4\}$  is not an  $n^t\eta$ -set in  $(J, \tau_R(P))$ .

**Theorem 3.2.** For a subset  $M$  of a space  $J$ , the following conditions are equivalent:

(i)  $M$  is  $n\alpha$ -closed.

(ii)  $M$  is an  $n^t\eta$ -set and  $n^*\mu_{\alpha}$ -closed.

*Proof.* (i)  $\rightarrow$  (ii) This is obvious.

(ii)  $\rightarrow$  (i) Since  $M$  is an  $n^t\eta$ -set, then using Theorem 3.1,  $M = S \cap n\alpha clo(M)$  where  $S$  is  $n^*$ gs- open in  $J$ . So,  $M \subseteq S$  and since  $M$  is  $n^*\mu_{\alpha}$ -closed, then  $n\alpha clo(M) \subseteq S$ . Therefore,  $n\alpha clo(M) \subseteq S \cap n\alpha clo(M) = M$ . Hence,  $M$  is  $n\alpha$ -closed. □

**Remark 3.5.** Intersection of two  $n^{\iota}\eta$ -sets is an  $n^{\iota}\eta$ -set.

**Remark 3.6.** Union of two  $n^{\iota}\eta$ -sets need not be an  $n^{\iota}\eta$ -set.

**Example 3.6.** Take  $J$  &  $n\tau_R(P)$  see Example 3.2. However, it is clear that  $\{2\}, \{4\}$  are  $n^{\iota}\eta$ -sets in  $(J, \tau_R(P))$  but their union  $\{2, 4\}$  is not an  $n^{\iota}\eta$ -set in  $(J, \tau_R(P))$ .

**Theorem 3.3.** For a subset  $M$  of a space  $J$ , the following conditions are equivalent.

- (i)  $M$  is  $n^*\mu_\alpha$ -open.
- (ii)  $M$  is an  $n^t\eta$ -set and  $n^*\mu_p$ -open.

*Proof.* Necessity: This is obvious.

Sufficiency: Assume that  $M$  is  $n^*\mu_p$ -open and an  $n^t\eta$ -set in  $J$ . Then  $M = S \cap G$  where  $S$  is  $n^*\mu_\alpha$ -open and  $G$  is a nt-set in  $J$ . Take  $H \subseteq M$ , where  $H$  is  $n^*$ gs-closed in  $J$ . Since  $M$  is  $n^*\mu_p$ -open in  $J$ ,  $H \subseteq \text{npinte}(M) = M \cap \text{ninte}(\text{nclo}(M)) = (S \cap G) \cap \text{ninte}[\text{nclo}(S \cap G)] \subseteq S \cap G \cap \text{ninte}(\text{nclo}(S)) \cap \text{ninte}(\text{nclo}(G)) = S \cap G \cap \text{ninte}(\text{nclo}(S)) \cap \text{ninte}(G)$ , since  $G$  is a nt-set. This implies,  $H \subseteq \text{ninte}(G)$ . Note that  $S$  is  $n^*\mu_\alpha$ -open and that  $H \subseteq S$ . So,  $H \subseteq n\alpha\text{inte}(S)$ . Therefore,  $H \subseteq n\alpha\text{inte}(S) \cap \text{ninte}(G) = n\alpha\text{inte}(M)$ . Hence  $M$  is  $n^*\mu_\alpha$ -open.  $\square$

#### 4. $n^t\eta$ -continuity & $n^t\eta$ -continuity

**Definition 4.1.** A map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  is called:

- (i) nA-continuous [10, 11] if  $i^{-1}(T)$  is an nA-set in  $J$  for each n-open  $T$  of  $L$ .
- (ii) nB-continuous [10, 11] if  $i^{-1}(T)$  is an nB-set in  $J$  for each n-open  $T$  of  $L$ .
- (iii)  $n\alpha$ -continuous [8] if  $i^{-1}(T)$  is an  $n\alpha$  open in  $J$  for each n-open  $T$  of  $L$ .
- (iv) n-LC-continuous [1] if  $i^{-1}(T)$  is an n-locally closed in  $J$  for each nano open  $T$  of  $L$ .
- (v)  $n\alpha$ B-continuous [4] if  $i^{-1}(T)$  is an  $n\alpha$ B-set in  $J$  for each n-open  $T$  of  $L$ .
- (vi)  $n\eta$ -continuous [4] if  $i^{-1}(T)$  is an  $n\eta$ -set in  $J$  for each n-open  $T$  of  $L$ .
- (vii)  $n^*\mu_\alpha$ -continuous [3] (respectively  $n^*\mu_p$ -continuous [3]) if  $i^{-1}(T)$  is an  $n^*\mu_\alpha$ -open (respectively  $n^*\mu_p$ -open) in  $J$  for each n-open  $T$  of  $L$ .

**Definition 4.2.** A map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  is called a  $n^t\eta$ -continuous (respectively  $n^t\eta$ -continuous) if  $i^{-1}(T)$  is an  $n^t\eta$ -set (respectively  $n^t\eta$ -set) in  $J$  for each n-open subset  $T$  of  $L$ .

**Definition 4.3.** A map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  is called a  $n^t\eta'$ -continuous if  $i^{-1}(T)$  is an  $n^t\eta'$ -set in  $J$  for each n-closed subset  $T$  of  $L$ .

**Remark 4.1.** It is clear that, a map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  is  $n\alpha$ -continuous iff  $i^{-1}(T)$  is an  $n\alpha$  closed set in  $J$  for each n-closed  $T$  of  $L$ .

**Proposition 4.1.** Every  $n\eta$ -continuous is  $n^t\eta$ -continuous.

*Proof.* Using Proposition 3.1.  $\square$

**Example 4.1.** Take  $J$  &  $n\tau_R(P)$  see Example 3.1. Take  $L = \{1, 2, 3, 4\}$  with  $L/R' = \{\{1\}, \{3\}, \{2, 4\}\}$  and  $Q = \{1, 2\}$ . Then  $n\tau'_R(Q) = \{\phi, \{1\}, \{2, 4\}, \{1, 2, 4\}, L\}$ . Define  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  be the identity map. However, it is  $n^t\eta$ -continuous but not  $n\eta$ -continuous, since  $i^{-1}(\{2, 4\}) = \{2, 4\}$  is not  $n\eta$ -set.

**Proposition 4.2.** Every  $n\alpha$ B-continuous is  $n^t\eta$ -continuous.

*Proof.* Using Proposition 3.2. □

**Example 4.2.** Take  $J = \{1, 2, 3\}$ , with  $J/R = \{\{3\}, \{1, 2\}, \{2, 1\}\}$  &  $P = \{1, 2\}$ . Then  $n\tau_R(P) = \{\phi, \{1, 2\}, J\}$ . Take  $L = \{1, 2, 3\}$  with  $L/R' = \{\{1\}, \{2, 3\}\}$  &  $Q = \{1\}$ . Then  $n\tau'_R(Q) = \{\phi, \{1\}, L\}$ . Then  $n^u\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}$  &  $n\alpha B$ -sets are  $\phi, J, \{3\}, \{1, 2\}$ . Define  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  be the identity map. However, it is  $n^u\eta$ -continuous but not  $n\alpha B$ -continuous, since  $i^{-1}(\{1\}) = \{1\}$  is not  $n\alpha B$ -set.

**Proposition 4.3.** Every  $n^* \mu_\alpha$ -continuous is  $n^u\eta$ -continuous.

*Proof.* Using Proposition 3.3. □

**Example 4.3.** Take  $J, n\tau_R(P)$  &  $i$  see Example 4.2. Take  $L = \{1, 2, 3\}$  with  $L/R' = \{\{3\}, \{1, 2\}\}$  &  $Q = \{3\}$ . Then  $n\tau'_R(Q) = \{\phi, \{3\}, L\}$ . Then  $n^u\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}$  &  $n^* \mu_\alpha$ -open sets are  $\phi, J, \{1\}, \{2\}, \{1, 2\}$ . Define  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  be the identity map. However, it is  $n^u\eta$ -continuous but not  $n^* \mu_\alpha$ -continuous, since  $i^{-1}(\{3\}) = \{3\}$  is not  $n^* \mu_\alpha$ -open set.

**Remark 4.2.** (i)  $n^* \mu_p$  continuity &  $n^u\eta$  continuity are independent.

(ii)  $n^* \mu_\alpha$  continuity &  $n^t\eta^t$  continuity are independent.

(iii)  $n^t\eta$  continuity &  $n^t\eta^t$  continuity are independent.

**Example 4.4.** Take  $J, n\tau_R(P), L, n\tau'_R(Y)$  &  $i$  see Example 4.3. Then  $n^u\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}$  &  $n^* \mu_p$ -open set are  $\phi, J, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$ . Define  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$  be the identity map. However, it is  $n^u\eta$ -continuous but not  $n^* \mu_p$ -continuous, since  $i^{-1}(\{3\}) = \{3\}$  is not  $n^* \mu_p$ -open.

**Example 4.5.** Take  $J = \{1, 2, 3\}$ , with  $J/R = \{\{2\}, \{1, 3\}, \{3, 1\}\}$  &  $P = \{1, 3\}$ . Then  $n\tau_R(P) = \{\phi, \{1, 3\}, J\}$ . Take  $L = \{1, 2, 3\}$  with  $L/R' = \{\{3\}, \{1, 2\}, \{2, 1\}\}$  &  $Q = \{1, 2\}$ . Then  $n\tau'_R(Q) = \{\phi, \{1, 2\}, L\}$ . Then  $n^u\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 3\}$  &  $n^* \mu_p$ -open sets are  $\phi, J, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$ . However, it is  $n^* \mu_p$ -continuous but not  $n^u\eta$ -continuous, since  $i^{-1}(\{1, 2\}) = \{1, 2\}$  is not  $n^u\eta$ -set.

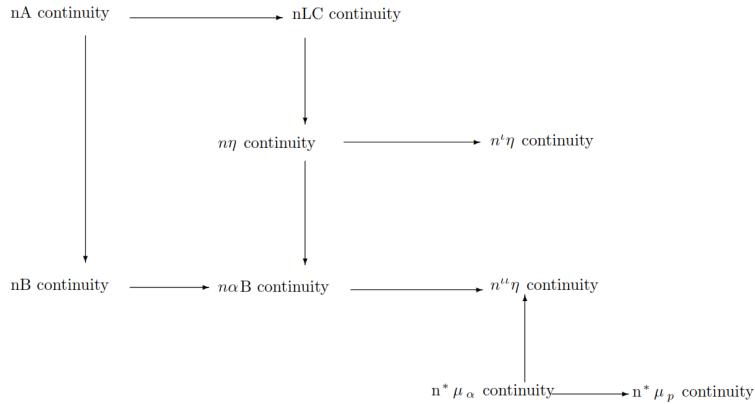
**Example 4.6.** Take  $J = \{1, 2, 3\}$ , with  $J/R = \{\{1\}, \{2, 3\}, \{3, 2\}\}$  &  $P = \{2, 3\}$ . Then  $n\tau_R(P) = \{\phi, \{2, 3\}, J\}$ . Take  $L, n\tau'_R(Q)$ , &  $i$  see Example 4.3. Then  $n^* \mu_\alpha$ -open sets are  $\phi, J, \{2\}, \{3\}, \{2, 3\}$  &  $n^t\eta$ -set are  $\phi, J, \{1\}, \{2\}, \{3\}, \{2, 3\}$ . However, it is  $n^* \mu_\alpha$ -continuous but not  $n^t\eta^t$ -continuous, since  $i^{-1}(\{1, 2\}) = \{1, 2\}$  is not  $n^t\eta$ -set.

**Example 4.7.** Take  $J, n\tau_R(P)$ , &  $i$  see Example 4.3. Take  $L = \{1, 2, 3\}$  with  $L/R' = \{\{1\}, \{2, 3\}, \{3, 2\}\}$  &  $Q = \{2, 3\}$ . Then  $n\tau'_R(Q) = \{\phi, \{2, 3\}, L\}$ . Then  $n^t\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}$  &  $n^* \mu_\alpha$ -open sets are  $\phi, J, \{1\}, \{2\}, \{1, 2\}$ . However, it is  $n^t\eta^t$ -continuous but not  $n^* \mu_\alpha$ -continuous, since  $i^{-1}(\{2, 3\}) = \{2, 3\}$  is not  $n^* \mu_\alpha$ -open.

**Example 4.8.** Take  $J, n\tau_R(P), L, n\tau'_R(Q)$  &  $i$  see Example 4.6. However, it is  $n^t\eta$ -continuous but not  $n^t\eta^t$ -continuous, since  $i^{-1}(\{1, 2\}) = \{1, 2\}$  is not  $n^t\eta$ -set.

**Example 4.9.** Take  $J, n\tau_R(P), L, n\tau'_R(Q)$  &  $i$  see Example 4.5. Then  $n^t\eta$ -sets are  $\phi, J, \{1\}, \{2\}, \{3\}, \{1, 3\}$ . However, it is  $n^t\eta^t$ -continuous but not  $n^t\eta$ -continuous, since  $i^{-1}(\{1, 2\}) = \{1, 2\}$  is not  $n^t\eta$ -set.

**Remark 4.3.** From the above discussions we obtain the following diagram where  $A \rightarrow B$  represents  $A$  implies  $B$ , but not conversely.



**Theorem 4.1.** Map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ , the following conditions are equivalent.

- (i)  $i$  is  $n\alpha$ -continuous.
- (ii)  $i$  is  $n^*\eta^*$ -continuous &  $n^* \mu_\alpha$ -continuous.

*Proof.* Using Definitions 4.1(7), 4.3, Remark 4.4 & Theorem 3.2, the proof is immediate. □

**Theorem 4.2.** Map  $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ , the following conditions are equivalent.

- (i)  $i$  is  $n^* \mu_\alpha$ -continuous.
- (ii)  $i$  is  $n''\eta$ -continuous &  $n^* \mu_p$ -continuous.

*Proof.* Using Theorem 3.3, the proof is immediate. □

**Acknowledgment**

The authors would like to thank the editors and the anonymous reviewers for their valuable comments and suggestions which have helped immensely in improving the quality of the paper.

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