Decomposition of $n\alpha$-continuity and $n^*\mu_\alpha$-continuity

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Abstract: The aim of this paper, we introduce the concepts of $n\eta$-sets, $n^{ii}\eta$-sets, $n\eta$-continuity, $n^{ii}\eta$-continuity & to find decomposition of $n\alpha$ continuity & $n^*\mu_\alpha$ continuity repectively in nano topological spaces.

Key words: $n\eta$-sets, $n\eta$-set, $n^{ii}\eta$-set, $n\eta$-continuity, $n^*\eta$-continuity & $n^{ii}\eta$-continuity

1. Introduction

Jayalakshmi and Janaki [5] introduced and studied the notions of nt-sets, nA-sets & nB-sets in nano topological spaces. Recently, Ganesan [4] introduced & studied $n\alpha$B-sets, $n\eta$-sets, $n\eta\zeta$-sets & to find a decomposition of nano continuity. In this paper, we introduce & study the notions of $n\eta$-sets, $n^{ii}\eta$-sets, $n\eta$-continuity, $n^{ii}\eta$-continuity & obtain decomposition of $n\alpha$ continuity & $n^*\mu_\alpha$ continuity. Moreover the study of $n\eta$-sets, $n^{ii}\eta$-sets led to some decomposition nano continuity are extensively developed and used in computer science & digital topology.

2. Preliminaries

Definition 2.1. [7]
If $(J, \tau_R(P))$ is the nano topological space with respect to $P$ where $P \subseteq J$ & if $M \subseteq J$, then

(i) The n-interior of the set $M$ is defined as the union of all n-open subsets contained in $M$ and it is denoted by $ninte(M)$. That is, $ninte(M)$ is the largest n-open subset of $M$.

(ii) The n-closure of the set $M$ is defined as the intersection of all n-closed sets containing $M$ and it is denoted by $nclo(M)$. That is, $nclo(M)$ is the smallest n-closed set containing $M$.

Definition 2.2. [7]
A subset $M$ of a space $(J, \tau_R(P))$ is called:

(i) $n\alpha$-closed if $nclo(ninte(nclo(M))) \subseteq M$.

(ii) n-semi-closed if $ninte(nclo(M)) \subseteq M$.

(iii) n-pre-closed if $nclo(ninte(M)) \subseteq M$.

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The complements of the above mentioned n-closed are called their respective n-open.

**Definition 2.3.** A subset $M$ of a space $(J, \tau_R(P))$ is called:

(i) a nt-set [5] if $\text{ninte(nclo}(M)) = \text{ninte}(M)$.

(ii) an nA-set [5] if $M = S \cap G$ where $S$ is n-open and $G$ is a n-regular-closed.

(iii) a nB-set [5] if $M = S \cap G$ where $S$ is n-open and $G$ is a nt-set.

(iv) a n-locally closed set [1] if $M = S \cap G$ where $S$ is n-open and $G$ is n-closed.

(v) an $n^\alpha$B-set [4] if $M = S \cap G$ where $S$ is n-open and $G$ is an $n^\alpha$-closed.

(vi) an $n\eta$-set [4] if $M = S \cap G$ where $S$ is n-open and G is an $n\alpha$-closed.

Collection of nt-sets (respectively nA-sets, nB-sets, n-locally closed sets, $n^\alpha$B-set, $n\eta$-set) in J is noted that nt(J) (respectively nA(J), nB(J), nLC(J), n$\alpha$B(J), n$\eta$(J)).

**Definition 2.4.** A subset $M$ of a space $(J, \tau_R(P))$ is called

(i) a $n^\hat{g}$-closed [6] if $\text{nclo}(M) \subseteq T$ whenever $M \subseteq T$ and $T$ is n-semi-open in $(J, \tau_R(P))$. The complement of $n^\hat{g}$-closed set is called $n^\hat{g}$-open.

(ii) n*gs-closed [2] if $\text{nsclo}(M) \subseteq T$ whenever $M \subseteq T$ and $T$ is $n^\hat{g}$-open in $(J, \tau_R(P))$. The complement of n*gs-closed set is called n*gs-open.

(iii) $n^*\mu_\alpha$-closed [2] if $\text{n\alpha clo}(M) \subseteq T$ whenever $M \subseteq T$ and $T$ is $n^*\text{gs}$-open in $(J, \tau_R(P))$. The complements of $n^*\mu_\alpha$-closed set is called $n^*\mu_\alpha$-open.

(iv) $n^*\mu_p$-closed [2] if $\text{npclo}(M) \subseteq T$ whenever $M \subseteq T$ and $T$ is $n^*\text{gs}$-open in $(J, \tau_R(P))$. The complements of $n^*\mu_p$-closed set is called $n^*\mu_p$-open.

**Proposition 2.1.** (i) Every n$\alpha$-open is $n^*\mu_\alpha$-open [2].

(ii) Every $n^*\mu_\alpha$-open is $n^*\mu_p$-open [2].

(iii) Every $n^*\mu_\alpha$-continuous is $n^*\mu_p$-continuous [3].

**Theorem 2.1.** (i) Every n-closed is Nt-set [5].

(ii) Every $n^\alpha$-closed is n-semi-closed [9].

(iii) Every nt-set is nB-set [5].

**Theorem 2.2.** [5]

(i) $M$ is nt-set iff it is n-semi-closed.

(ii) Intersection two nt-sets is also a nt-set.
3. \(n^r\eta\)-sets & \(n^s\eta\)-sets

**Definition 3.1.** A subset \(M\) of a space \(J\) is called

(i) an \(n^r\eta\)-set if \(M = S \cap G\) where \(S\) is \(n^s\text{gs-open}\) and \(G\) is \(n\alpha\)-closed.

(ii) an \(n^s\eta\)-set if \(M = S \cap G\) where \(S\) is \(n^s\mu_\alpha\)-open and \(G\) is a \(n\eta\)-set.

Collection of all \(n^r\eta\)-sets (respectively \(n^s\eta\)-sets) in \(J\) will be note that \(n^r\eta(J)\) (respectively \(n^s\eta(J)\)).

**Proposition 3.1.** Every \(n\eta\)-set is \(n^r\eta\)-set.

*Proof.* Take \(E\) be \(n\eta\)-set. Then \(E = S \cap G\), where \(S\) is \(n\)-open and \(G\) is \(n\alpha\)-closed. Since every \(n\)-open is \(n^s\text{gs}\) open, \(S\) is \(n^s\text{gs}\) open. Hence \(E\) is \(n^r\eta\)-set.

**Example 3.1.** Take \(J = \{1, 2, 3, 4\}\) with \(J/ R = \{\{3\}, \{4\}, \{1, 2\}\}\) & \(P = \{2\}\). The \(n\tau_R(P) = \{\phi, \{1, 2\}, J\}\). Then \(n^r\eta\)-sets are \(\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}\) & \(n\eta\)-set are \(\phi, J, \{3\}, \{4\}, \{1, 2\}, \{3, 4\}\). However, it is clear that \(\{1, 3\}\) is \(n^r\eta\)-set but it is not \(n\eta\)-set.

**Proposition 3.2.** Every \(n\alpha\) \(B\)-set is \(n^s\eta\)-set.

*Proof.* Take \(E\) be \(n\alpha\) \(B\)-set. Then \(E = S \cap G\), where \(S\) is \(n\alpha\)-open and \(G\) is \(n\text{t}\)-set. Since every \(n\alpha\)-open set is \(n^s\mu_\alpha\)-open, \(S\) is \(n^s\mu_\alpha\)-open. Hence \(E\) is \(n^s\eta\)-set.

**Example 3.2.** Take \(J\) & \(n\tau_R(P)\) see Example 3.1. Then \(n^s\eta\)-set are \(\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}\). However, it is clear that \(\{1, 3\}\) is \(n^s\eta\)-set but it is not \(n\alpha\) \(B\)-set.

**Proposition 3.3.** Every \(n^s\mu_\alpha\)-open set is \(n^s\eta\)-set.

*Proof.* Using Definitions 2.4(iii) and 3.1(ii).

**Example 3.3.** Take \(J\) & \(n\tau_R(P)\) see Example 3.2. Then \(n^s\mu_\alpha\)-open are \(\phi, J, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}\). It is clear that \(\{3, 4\}\) is \(n^s\eta\)-set but it is not \(n^s\mu_\alpha\)-open.

**Remark 3.1.** (i) \(n^r\eta\)-sets & \(n^s\mu_\alpha\)-closed are independent.
(ii) \(n^s\eta\)-sets & \(n^s\mu_\beta\)-closed are independent.

**Example 3.4.** (i) Take \(J\) & \(n\tau_R(P)\) see Example 3.1. Then \(n^s\mu_\alpha\)-closed are \(\phi, J, \{3\}, \{4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\). However, it is clear that \(\{1, 3, 4\}\) is \(n^s\mu_\alpha\)-closed but not \(n^r\eta\)-set & also it is clear that \(\{1, 2, 3\}\) is \(n^r\eta\)-set but not \(n^s\mu_\alpha\)-closed in \((J, \tau_R(P))\).
(ii) Let \(J\) & \(n\tau_R(P)\) see Example 3.2. Then \(n^s\mu_\beta\)-closed are \(\phi, J, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\). However, it is clear that \(\{1, 3\}\) is \(n^s\mu_\beta\)-closed but not \(n^s\eta\)-set & also it is clear that \(\{1, 2, 4\}\) is \(n^s\eta\)-set but not \(n^s\mu_\beta\)-closed in \((J, \tau_R(P))\).

**Remark 3.2.** We discuss above results see the diagram.

where none of these implications is reversible as shown by [4].

**Theorem 3.1.** For a subset \(M\) of a space \(J\), the following conditions are equivalent.
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(i) M is an \(n^{*}\eta\)-set.

(ii) \(M = S \cap \text{naclo}(M)\) for some \(n^{*}\text{gs-open}\ S\).

**Proof.** (i) \(\rightarrow\) (ii) Since M is an \(n^{*}\eta\)-set, then \(M = S \cap G\), where S is \(n^{*}\text{gs-open}\) and G is \(n\alpha\) closed. So, \(M \subseteq S\) and \(M \subseteq G\). Hence \(\text{naclo}(M) \subseteq \text{naclo}(G)\). Therefore \(M \subseteq S \cap \text{naclo}(M) \subseteq S \cap \text{naclo}(G) = S \cap G = M\). Thus, \(M = S \cap \text{naclo}(M)\).

(ii) \(\rightarrow\) (i) It is obvious because \(\text{naclo}(M)\) is \(n\alpha\)-closed. (Since M is \(n\alpha\)-closed iff \(M = \text{naclo}(M)\)). \(\square\)

**Remark 3.3.** Intersection of two \(n^{*}\eta\)-sets is an \(n^{*}\eta\)-set.

**Remark 3.4.** Union of two \(n^{*}\eta\)-sets need not be an \(n^{*}\eta\)-set.

**Example 3.5.** Take \(J \& \tau_{R}(P)\) see Example 3.1. However, it is clear that \(\{1, 3\}, \{4\}\) are \(n^{*}\eta\)-sets in \((J, \tau_{R}(P))\) but their union \(\{1, 3, 4\}\) is not an \(n^{*}\eta\)-set in \((J, \tau_{R}(P))\).

**Theorem 3.2.** For a subset \(M\) of a space \(J\), the following conditions are equivalent:

(i) \(M\) is \(n\alpha\)-closed.

(ii) \(M\) is an \(n^{*}\eta\)-set and \(n^{*}\mu_{\alpha}\)-closed.

**Proof.** (i) \(\rightarrow\) (ii) This is obvious.

(ii) \(\rightarrow\) (i) Since M is an \(n^{*}\eta\)-set, then using Theorem 3.1, \(M = S \cap \text{naclo}(M)\) where S is \(n^{*}\text{gs-open}\) in J. So, \(M \subseteq S\) and since M is \(n^{*}\mu_{\alpha}\)-closed, then \(\text{naclo}(M) \subseteq S\). Therefore, \(\text{naclo}(M) \subseteq S \cap \text{naclo}(M) = M\). Hence, \(M\) is \(n\alpha\)-closed. \(\square\)

**Remark 3.5.** Intersection of two \(n^{*}\eta\)-sets is an \(n^{*}\eta\)-set.

**Remark 3.6.** Union of two \(n^{*}\eta\)-sets need not be an \(n^{*}\eta\)-set.

**Example 3.6.** Take \(J \& \tau_{R}(P)\) see Example 3.2. However, it is clear that \(\{2\}, \{4\}\) are \(n^{*}\eta\)-sets in \((J, \tau_{R}(P))\) but their union \(\{2, 4\}\) is not an \(n^{*}\eta\)-set in \((J, \tau_{R}(P))\).
Theorem 3.3. For a subset $M$ of a space $J$, the following conditions are equivalent.

(i) $M$ is $n^*\mu_\alpha$-open.

(ii) $M$ is an $n^\eta$-set and $n^*\mu_p$-open.

Proof. Necessity: This is obvious.

Sufficiency: Assume that $M$ is $n^*\mu_p$-open and an $n^\eta$-set in $J$. Then $M = S \cap G$ where $S$ is $n^*\mu_\alpha$-open and $G$ is a $n^\eta$-set in $J$. Take $H \subseteq M$, where $H$ is $n^*gs$-closed in $J$. Since $M$ is $n^*\mu_p$-open in $J$, $H \subseteq npinte(M) = M \cap ninte(\text{ncl}(M)) = (S \cap G) \cap ninte(\text{ncl}(S)) \cap ninte(\text{ncl}(G)) = S \cap G \cap ninte(\text{ncl}(S)) \cap ninte(G)$, since $G$ is a $n^\eta$-set. This implies, $H \subseteq ninte(G)$. Note that $S$ is $n^*\mu_\alpha$-open and that $H \subseteq S$. So, $H \subseteq n^\alpha inte(S)$. Therefore, $H \subseteq n^\alpha inte(S) \cap ninte(G) = n^\alpha inte(M)$. Hence $M$ is $n^*\mu_\alpha$-open.

4. $n^\eta$-continuity & $n^*\eta$-continuity

Definition 4.1. A map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ is called:

(i) $nA$-continuous [10, 11] if $i^{-1}(T)$ is an $nA$-set in $J$ for each $n$-open $T$ of $L$.

(ii) $nB$-continuous [10, 11] if $i^{-1}(T)$ is an $nB$-set in $J$ for each $n$-open $T$ of $L$.

(iii) $n^\alpha$-continuous [8] if $i^{-1}(T)$ is an $n^\alpha$-open in $J$ for each $n$-open $T$ of $L$.

(iv) $n$-LC-continuous [1] if $i^{-1}(T)$ is an $n$-locally closed in $J$ for each nano open $T$ of $L$.

(v) $n^\alpha B$-continuous [4] if $i^{-1}(T)$ is an $n^\alpha B$-set in $J$ for each $n$-open $T$ of $L$.

(vi) $n^\eta$-continuous [4] if $i^{-1}(T)$ is an $n^\eta$-set in $J$ for each $n$-open $T$ of $L$.

(vii) $n^*\mu_\alpha$-continuous [3] (respectively $n^*\mu_p$-continuous [3]) if $i^{-1}(T)$ is an $n^*\mu_\alpha$-open (respectively $n^*\mu_p$-open) in $J$ for each $n$-open $T$ of $L$.

Definition 4.2. A map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ is called a $n^\eta$-continuous (respectively $n^*\eta$-continuous) if $i^{-1}(T)$ is an $n^\eta$-set (respectively $n^*\eta$-set) in $J$ for each $n$-open subset $T$ of $L$.

Definition 4.3. A map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ is called a $n^\eta'$-continuous if $i^{-1}(T)$ is an $n^\eta'$-set in $J$ for each $n$-closed subset $T$ of $L$.

Remark 4.1. It is clear that, a map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ is $n^\alpha$-continuous iff $i^{-1}(T)$ is an $n^\alpha$ closed set in $J$ for each $n$-closed $T$ of $L$.

Proposition 4.1. Every $n^\eta$-continuous is $n^\eta'$-continuous.


Example 4.1. Take $J \& n^\eta R(P)$ see Example 3.1. Take $L = \{1, 2, 3, 4\}$ with $L/R' = \{\{1\}, \{3\}, \{2, 4\}\}$ and $Q = \{1, 2\}$. Then $n^\eta R(Q) = \{\phi, \{1\}, \{2, 4\}, \{1, 2, 4\}, L\}$. Define $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$ be the identity map. However, it is $n^\eta$-continuous but not $n^\eta$-continuous, since $i^{-1}(\{2, 4\}) = \{2, 4\}$ is not $n^\eta$-set.

Proposition 4.2. Every $n^\alpha B$-continuous is $n^*\eta$-continuous.
Example 4.2. Take \( J = \{1, 2, 3\} \), with \( J/ R = \{\{\emptyset\}, \{1, 2\}, \{2, 3\}\} \) & \( P = \{1, 2\} \). Then \( \nu \tau_R(P) = \{\phi, \{1, 2\}\} \). Take \( L = \{1, 2, 3\} \) with \( L/ R' = \{\{1\}, \{2, 3\}\} \) & \( Q = \{1\} \). Then \( \nu \tau_R'(Q) = \{\phi, \{1\}, L\} \). Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\} \) & \( \eta B\)-sets are \( \phi, J, \{3\}, \{1, 2\} \). Define \( i : (J, \tau_R(P)) \rightarrow (L, \tau_R'(Q)) \) be the identity map. However, it is \( \nu^* \eta\)-continuous but not \( \eta B\)-continuous, since \( i^{-1}(\{1\}) = \{1\} \) is not \( \eta B\)-set.

Proposition 4.3. Every \( \nu^* \mu_\alpha\)-continuous is \( \nu^* \eta\)-continuous.

Example 4.3. Take \( J, \nu \tau_R(P) \) & \( i \) see Example 4.2. Take \( L = \{1, 2, 3\} \) with \( L/ R' = \{\{\emptyset\}, \{1, 2\}\} \) & \( Q = \{3\} \). Then \( \nu \tau_R(Q) = \{\phi, \{3\}\} \). Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\} \) & \( \nu^* \mu_\alpha\)-open sets are \( \phi, \{1\}, \{2\}, \{3\}, \{1, 2\} \). Define \( i : (J, \tau_R(P)) \rightarrow (L, \tau_R'(Q)) \) be the identity map. However, it is \( \nu^* \eta\)-continuous but not \( \nu^* \mu_\alpha\)-continuous, since \( i^{-1}(\{1\}) = \{3\} \) is not \( \nu^* \mu_\alpha\)-open set.

Remark 4.2. (i) \( \nu^* \mu_\alpha\) continuity & \( \nu^* \eta\) continuity are independent.
(ii) \( \nu^* \mu_\alpha\) continuity & \( \nu^* \eta\) continuity are independent.
(iii) \( \eta\) continuity & \( \nu^* \eta\) continuity are independent.

Example 4.4. Take \( J, \nu \tau_R(P), L, \nu \tau_R'(Y) \) & \( i \) see Example 4.3. Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\} \). Define \( i : (J, \tau_R(P)) \rightarrow (L, \tau_R'(Q)) \) be the identity map. However, it is \( \nu^* \eta\)-continuous but not \( \nu^* \mu_\alpha\)-continuous, since \( i^{-1}(\{3\}) = \{3\} \) is not \( \nu^* \mu_\alpha\)-open.

Example 4.5. Take \( J = \{1, 2, 3\} \), with \( J/ R = \{\{\emptyset\}, \{1, 3\}, \{3, 1\}\} \) & \( P = \{1, 3\} \). Then \( \nu \tau_R(P) = \{\phi, \{1, 3\}\} \). Take \( L = \{1, 2, 3\} \) with \( L/ R' = \{\{1\}, \{2, 3\}\} \) & \( Q = \{1, 2\} \). Then \( \nu \tau_R'(Q) = \{\phi, \{1, 2\}\} \). Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 3\}\) & \( \nu^* \mu_\alpha\)-open sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\} \). However, it is \( \nu^* \mu_\alpha\)-continuous but not \( \nu^* \eta\)-continuous, since \( i^{-1}(\{1\}) = \{1, 2\} \) is not \( \nu^* \eta\)-set.

Example 4.6. Take \( J = \{1, 2, 3\} \), with \( J/ R = \{\{\emptyset\}, \{1, 3\}, \{3, 2\}\} \) & \( P = \{2, 3\} \). Then \( \nu \tau_R(P) = \{\phi, \{2, 3\}\} \). Take \( L, \nu \tau_R'(Q), \) & \( i \) see Example 4.3. Then \( \nu^* \mu_\alpha\)-open sets are \( \phi, J, \{2\}, \{3\}, \{2, 3\} \) & \( \nu^* \eta\)-set are \( \phi, J, \{1\}, \{2\}, \{3\}, \{2, 3\} \). However, it is \( \nu^* \mu_\alpha\)-continuous but not \( \nu^* \eta\)-continuous, since \( i^{-1}(\{1\}) \) \( \{1, 2\} \) is not \( \nu^* \eta\)-set.

Example 4.7. Take \( J, \nu \tau_R(P), \) & \( i \) see Example 4.3. Take \( L = \{1, 2, 3\} \) with \( L/ R' = \{\{\emptyset\}, \{1, 3\}, \{3, 2\}\} \) & \( Q = \{2, 3\} \). Then \( \nu \tau_R'(Q) = \{\phi, \{2, 3\}, L\} \). Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 2\} \) & \( \nu^* \mu_\alpha\)-open sets are \( \phi, J, \{1\}, \{2\}, \{1, 2\} \). However, it is \( \nu^* \eta\)-continuous but not \( \nu^* \mu_\alpha\)-continuous, since \( i^{-1}(\{2, 3\}) = \{2, 3\} \) is not \( \nu^* \mu_\alpha\)-open.

Example 4.8. Take \( J, \nu \tau_R(P), L, \nu \tau_R'(Q) \) & \( i \) see Example 4.6. However, it is \( \nu^* \eta\)-continuous but not \( \nu^* \eta\)-continuous, since \( i^{-1}(\{1, 2\}) = \{1, 2\} \) is not \( \nu^* \eta\)-set.

Example 4.9. Take \( J, \nu \tau_R(P), L, \nu \tau_R'(Q) \) & \( i \) see Example 4.5. Then \( \nu^* \eta\)-sets are \( \phi, J, \{1\}, \{2\}, \{3\}, \{1, 3\}\). However, it is \( \nu^* \eta\)-continuous but not \( \nu^* \eta\)-continuous, since \( i^{-1}(\{1, 2\}) = \{1, 2\} \) is not \( \nu^* \eta\)-set.
Remark 4.3. From the above discussions we obtain the following diagram where $A \rightarrow B$ represents $A$ implies $B$, but not conversely.

Theorem 4.1. Map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$, the following conditions are equivalent.

(i) $i$ is $n\alpha$-continuous.

(ii) $i$ is $n^*\eta^*$-continuous & $n^*\mu_\alpha$-continuous.

Proof. Using Definitions 4.1(7), 4.3, Remark 4.4 & Theorem 3.2, the proof is immediate.

Theorem 4.2. Map $i : (J, \tau_R(P)) \rightarrow (L, \tau'_R(Q))$, the following conditions are equivalent.

(i) $i$ is $n^*\mu_\alpha$-continuous.

(ii) $i$ is $n^*\eta$-continuous & $n^*\mu_p$-continuous.

Proof. Using Theorem 3.3, the proof is immediate.

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