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# Mathematical Proficiency of Pre-service Secondary School Teachers in Triangle Trigonometry in Nigeria

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**Abstract:** This paper reported the assessment as well as analysis of the mathematical proficiency of pre-service teachers of junior/senior secondary school in Nigeria in triangle trigonometry. It accounted for the three strands of mathematical proficiency, i.e. conceptual understanding, procedural fluency, and strategic competence. Mixed methods research design was used for the study. It involves collection and analysis of data quantitatively and qualitatively. Quantitative data were collected using a test instrument designed by the researcher with an alpha reliability (Cronbach's Alpha) of 0.64. Similarly, qualitative data were generated by means of content analysis of the pre-service teachers' test manuscripts. The pre-service teachers involved in the study were final year students in the Mathematics department of a Federal College of Education in the north western region of Nigeria. The outcome showed that the pre-service teachers had a weak understanding of basic trigonometric ratios, in addition to the sine and cosine rules in both isolated and non-isolated cases. Equally, their procedural fluency along with strategic competence was proven to be murky. The general outcome of the study was that the pre-service teachers were not mathematically proficient in triangle trigonometry.

**Key words:** Mathematical proficiency, Conceptual understanding, Procedural fluency, Strategic competence, Trigonometric ratios of sine and cosine

## 1. Introduction

There has been growing concern every year the National examination results such as West African Senior Secondary Certificate Examination, (WASSCE); National Examination Commission, (NECO); and Unified Tertiary Matriculation Examination, (UTME) in Nigeria are released. Students' poor performance in Mathematics in these examinations was one area that attracts a lot of denigration. The causes of this weak performance with particular reference to mathematics have generated much debate, mostly bidirectional. One group put the cause of low achievement in mathematics on the students, whereas the other, tended to be disposed towards issues associated with teaching and learning mathematics nationwide. The issues of teaching and learning of mathematics are multifaceted and include, but are not limited to, curriculum (structure, implementation, quality); assessment; availability of qualified and well trained mathematics teachers, and so on. Students' success in mathematics is greatly impacted by the quality of their teachers. In an attempt to improve students' success in mathematics,[29] , states that, in 2007, the Nigerian Educational Research and Development Council [21] reviewed the secondary (senior/junior) school Mathematics curriculum to give mathematics education a purposeful approach. In a similar vein, in 2012, the National Commission for Colleges of Education in Nigeria,[23] has reviewed the teacher education curriculum (mathematics teacher education inclusive) to train professional


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teachers who have a sound and critical understanding of the mathematics curriculum content, [24]. The National Commission for Colleges of Education is an umbrella body saddled with the responsibility of laying down minimum standards for all *primary/junior/senior* secondary school teacher education programs as well as accrediting their certificates. Even though previous endeavors to improve mathematics teaching and learning dwelt mostly on students' success, example NERDC project; the most recent efforts have focused attention on enhancing the quality of teachers, for example, National Commission for Colleges of Education, [23]. There were no researches that document attainment of the anticipated qualities of mathematics teachers in Nigeria, but a small number that existed, example [29], have shown that "pre-service mathematics teachers were unable to identify words/phrases that represent mathematical operations (addition, subtraction, multiplication, division and equality) fluently in isolated cases, as well as construct algebraic expressions, equations and their solutions from word problems", $(p.350)$. This has indicated that pre-service mathematics teachers were poorly prepared at least in algebraic processing, particularly algebraic word problems.

To further buttress the bad state of teaching and learning of mathematics in Nigeria, recently, the Director-General and Chief Executive officer, National Mathematical Centre, Abuja Professor Adewale Solarin at a press briefing in Abuja to announce the international symposium on "Current trends in mathematical science teachers and applications" organized by African Academy of Sciences, AAS, and African Mathematical Union, AMU, holding May $17, 2016$, stated that: ... " [*the*] dearth of teachers for mathematics-related subjects was partly responsible for the phobia and poor performance of students in mathematics in public examination... urged the Federal Government to declare a state of emergency on mathematics", [30]. The fundamental question is: How can we fix this problem? The ideal thing to do is to train competent mathematics teachers at all levels that are mathematically proficient. This leads to another question. What is mathematical proficiency? The intent of this paper is not to discuss in depth the meaning of mathematical proficiency, but rather use its descriptive structure to assess the competency and proficiency level of pre-service mathematics teachers at junior/senior secondary school levels in Nigeria. These pre-service teachers are about to graduate with the National Certificate in Education (NCE). Specifically, the study focuses on investigating the mathematical proficiency of pre-service teachers in triangle trigonometry. Triangle trigonometry is an important component of the junior/senior secondary school mathematics curriculum in Nigeria. Generally, trigonometry has an important place in the mathematics curriculum in many countries even though its meaning, value and significance may change from country to country at secondary school, [10].

## 2. Theoretical framework

This study was designed based on the Mathematical proficiency framework, [20]. Mathematical proficiency provides a description of cognitive changes that are expected in children that would help them succeed in mathematics, (Adding it up, helping children learn mathematics). It constitutes five strands that feature three mathematical abilities (conceptual understanding, procedural knowledge and problem solving) as well as additional specifications for reasoning, connections, and communication, [20]. The five strands are interwoven and interdependent in the development of proficiency in mathematics, and are given as follows:

- Conceptual understanding: ... refers to an integrated and functional grasp of mathematical ideas, $(p.118)$.
- Procedural fluency: ... refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently, $(p.121)$.

- Strategic competence ... refers to the ability to formulate mathematical problems, represent them, and solve them, $(p.124)$.
- Adaptive reasoning ... refers to the capacity to think logically about the relationships among concepts and situations, (129).
- Productive disposition ... refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics, $(p.131)$.

Several studies have supported the importance for teacher knowledge of subject matter in teacher education programs. It was even suggested [14] "that teachers should develop proficiency in mathematics, such as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition [20], reported, [18]. In addition to what was alluded above, [19] holds the notion that teachers should attain a deep, vast and thorough understanding of the mathematical topics beyond simply knowing the content that they teach. The essence is that they can use their knowledge of facts, procedures, definitions and concepts fluently. The interwoven nature of the mathematical proficiency strands, coupled with its furnishing potential, makes it a convenient tool to assess and analyze the mathematical understanding and fluency (mathematical proficiency) of pre-service teachers for *junior/senior* secondary schools in Nigeria. The current study was designed based on the first three mathematical proficiency strands stated above.

## 3. Purpose of the Study

Trigonometry is among the five strands of Mathematics curriculum in Nigeria [21]. The content covered at the secondary school level in Nigeria is mostly triangle trigonometry, even though graphs of trigonometric functions are covered at senior secondary school. Its importance is well articulated, cutting across many applications in engineering, physics, astronomy, architecture and so on, [11]. Specifically, according to [21] mathematics education curriculum, the performance objectives of teaching trigonometric strand at junior/senior secondary schools in Nigeria are to acquaint students so that they should be able to: (1) solve right-angle triangle using trigonometric functions of sine, cosine and tangent without calculators. (2) Relate sine and cosine ratios to the unit circle. (3) Draw the graphs of sine and cosine functions and their transformation. (4) Solve non-right angle triangle using sine and cosine rules. (5) Solve practical problems related to angles elevation/depression using (sine/cosine/tangent functions) as well as bearing problems using $(sine/cosinerules)$. Mixed method research design was used in this study. The researcher adopted, specifically, sequential explanatory mixed methods to explore mathematical proficiency of pre-service teachers in triangle trigonometry. The intent of this design is to pull together and examine data quantitatively (descriptive statistics) and then follow-up with qualitative content analysis of students' manuscripts, [9]. The first part i.e. quantitative component, would allow easy categorization of the sample; whereas the qualitative section would give further supplementary elucidation of their attributes in more particularized form, [28]. Research questions guiding this study are:

- What kind of knowledge of basic trigonometry ratios of right angle triangle did pre-service teachers possess?
- Are the pre-service teachers able to solve right-angle triangle problems?
- Are the pre-service teachers able to recall and state sine and cosine rules?
- What is the pre-service teachers' ability in solving non-right angle triangle problems?

## 4. Related literature

Researches over the years have shifted attention towards investigating what teachers have known, because of the belief that knowledge matters. Subject matter knowledge of the teachers is considered an indispensable component of any teacher education program [3]; [2]; [22]; as cited by [18]. Furthermore, reiterating the importance of subject-matter knowledge, [27]; [15]; cited by [25] states that "teachers were unsuccessful in promoting mathematical learning outside of the limits of their own understanding and their knowledge was significantly related to student achievement gains". Moreover, [26]; as well as [15] claimed that pre-service teachers' subject matter knowledge of different mathematics concepts was found to be significantly better than their syntactic knowledge. Invariably, this affected the way they taught mathematics. Researches related to subject matter knowledge of mathematics teachers in Nigeria, specifically, found that trigonometry is scanty, despite its importance in the mathematics curriculum. Globally, there existed documented difficulties associated with learning and understanding of trigonometry. For example, learning and understanding difficulties associated with trigonometric functions [31]; [11]; [16]; [7]; [4]; difficulty transiting from right angle triangle trigonometry to unit circle trigonometry, for example, [5]. There are also studies related to structures of trigonometric textbooks, for example, [6], and so on.

### 4.1. Trigonometric functions

Researchers have shown that the idea of expressing the trigonometric functions of sine, cosine and tangent as functions was difficult, [31]. In his study involving two groups of college students, one group was taught trigonometry using the traditional method trigonometric operations are first taught as ratios, for example, sine is defined as in a right-angled triangle, Weber, [32], whereas the second group was taught using [**?** ] procept (an amalgam of three components: a process which produces a mathematical object, and a symbol which is used to represent either process or object", $pg.120$) theories. The outcome revealed that the group that was taught by [**?** ] premise comprehended trigonometric functions better. In a similar vein, [11] states that, [16] reported that students are having difficulties conceptualizing trigonometric functions in the domain of real numbers. Citing examples from [16], [11], states that [16] reported, that

> 21% of the students claimed that $f(x) = sinx$ means the ratio of opposite to hypotenuse, whereas, 60% could not define $y = sinx$ as a function.

### 4.2. Triangle trigonometry

Introductory trigonometry was traditionally taught at junior/senior secondary schools using right angle triangle in many countries, for example, in Nigeria [21], and The Netherlands [11]. There are researches that support this approach despite its setback. [17] conducted a study comparing teaching trigonometry using ratio of right-triangle and unit circle methods. It was reported that there is significant proof that indicated that the ratio method of teaching introductory trigonometry was better than the unit circle method. Furthermore, the retention level of the skills acquired was high over a period of time, consequently, this created a positive viewpoint towards trigonometry as well as mathematics in general. In a similar vein, [32] states that triangle trigonometry allows students to solve vector problems in physics and other related problems; while, [34], reported that students' understanding of trigonometric ratios can be enhanced using the historical approach.

### 4.3. Textbooks treatment of trigonometry

Students in Nigeria start learning trigonometry for the first time at junior secondary school as trigonometric ratios of a right-angle triangle, but they are introduced to non-right angle triangle trigonometry, unit circle trigonometry and graphs of trigonometric functions at a much later stage of their secondary education, i.e. senior secondary school. Most, if not all, secondary school mathematics text books in Nigeria follow the same structure. Trigonometry is connected with algebra, geometry, visualization as well as calculus (differential and integral), vector analysis and so on. For those students who intend to study mathematics at higher level it serves as a good background to help them succeed. Based on the aforementioned facts, it is pertinent to explore the extent to which pre-service teachers would be able to teach trigonometry effectively at junior and secondary school.

## 5. Method

### 5.1. Design

The design used for this study is a sequential mixed method, [9], cited by [29]. It is a method of inquiry that comprises the techniques of collecting data quantitatively (test instrument) and qualitatively (content/document analysis) sequentially. The two methods are employed together consecutively because of the desire to give a detailed explanation and enhance the characteristics of the outcome, [8]. Furthermore, the two methods would enhance the data collected, which consequently facilitates in the discovery of errors and misunderstandings with respect to some basic definitions of trigonometric ratios in a right-angled triangle. In addition to that, pre-service teachers' ability or otherwise in the solutions of non-right angle triangles would be analyzed. The results from the two methods would reveal the infirmity or robustness of the pre-service teachers' content knowledge in triangle trigonometry.

### 5.2. Demographic information of the Participants

The number of participants in the study was fifty nine, consisting of forty two (71%) male and seventeen (29%) females. All the participants have completed almost all courses required leading to the award of Nigeria Certificate in Education (NCE) program. Earning this credential would qualify them to teach at primary, junior and senior secondary school in Nigeria.

### 5.3. Instrument

The test instrument was designed by the researcher to actualize the objective of the study. The reliability test of the instrument used for data collection was Cronbach's Alpha of $0.64(\alpha = 0.64)$. This reliability Alpha value is quite high and acceptable. The instrument consists of four major components: (1) the assessment of the basic knowledge of trigonometric ratios in right angled triangle; (2) the ability to solve right-angled triangle problems; (3) the assessment of basic knowledge of sine and cosine formulas; (4) the ability to apply sine and cosine formulas to solve non-right angled triangle problems. The first and third components measure pre-service basic skills and knowledge of right-angled and non-right angled triangles, whereas the second and fourth sections assess applications of those fundamental concepts to right-angled and non-right-angled triangle trigonometric problems. A scoring rubric was designed based on Item Response Theory (IRT), nondichotomous model [33]. Solutions were grouped into three using grading principles given below. Each student's manuscript was rated on a continuous scale from 0 to 20 points.

**Table 1**. Grading Rubric.

| | Grading Principle |
|---|---|
| Correct solution | This consists of knowledge of correct concepts, using procedures accurately and efficiently, the ability to formulate mathematical problems properly and solve them to get correct answers. |
| Partially correct solution | It consists of either correct knowledge of concepts or use of acceptable procedures or the ability to construct mathematical problems, but being unable to solve them. |
| Incorrect solution | The Inability to either state/recall correct concepts, to apply appropriate procedures, to formulate mathematical problem and solve it; or just leaving a blank space. |

## 6. Analysis

The purpose of this paper is to measure pre-service content knowledge of triangle trigonometry (right and non-right triangle trigonometry). Specifically, it is to explore and assess basic knowledge of concepts and procedures of triangle trigonometry, formulate and solve mathematical problems involving right and non-right triangle trigonometric problems. To assess these capabilities and qualities, data were collected using test instrument designed by the researcher based on seven variables and are: (1) Basic knowledge of trigonometric ratios, (BKTR); (2) Apply basic knowledge of trigonometric ratios, (ABKTR); (3) Visualize right triangle trigonometric problems, (VRTP); (4) Solve right triangle trigonometric problems, (SRTP); (5) Basic knowledge of non-right angle triangles, (BKNRT); (6) Visualize non-right angle triangle problems, (VNTRP); (7) Solve non-right angle triangle problems, (SNRTP). The data collected were analyzed quantitatively and qualitatively below.

**Table 2**. Descriptive Statistics of all Variables.

| Variables | Mean | Median | Mode | Std. Dev | Maximum |
|---|---|---|---|---|---|
| BKTR* | 4.11 | 5 | 6 | 2.27 | 6 |
| ABKTR* | 4.31 | 1 | 0 | 6.06 | 20 |
| VRTP* | 2.2 | 2.5 | 2.5 | 1.72 | 5 |
| SRTP* | 0.7 | 0 | 0 | 1.48 | 5 |
| BKNRT* | 1.33 | 1 | 0 | 1.53 | 4 |
| VNRTP* | 2.41 | 2 | 5 | 2.08 | 5 |
| SNRTP* | 0 | 0 | 0 | 0 | 15 |

$BKTR* =$ Basic knowledge of trigonometric ratio, $ABKTR* =$ Apply basic knowledge of trigonometric ratio. $VRTP* =$ Visualize right triangle trigonometric problem, $SRTP* =$ Solve right triangle trigonometric problem. $BKNRT* =$ Basic knowledge of non-right angle triangle, $VNRTP* =$ Visualize non-right angle triangle problem. SNRTP* = Solve non-right angle triangle problem.

### 6.1. Research question 1:

What kind of knowledge of basic trigonometry ratios of right angle triangle did pre-service teachers possess?

This research question was set up to assess whether the pre-service teachers are able to recall and state basic knowledge of trigonometric ratios. It uses the variable BKTR (Basic knowledge of trigonometric ratio). It was expected that they would be able to successfully accomplish this task fluently. The reason for this

anticipation was that they have finished a course in trigonometry (Course title: Trigonometry, Course code: $MATH112$). This course is contained in regular NCE Mathematics curriculum as a core course. Similarly, trigonometry was covered extensively in Junior and Senior Secondary school Mathematics curriculum (NERDC). Quantitatively, the descriptive statistics for this variable (BKTR) which evaluate the research question above as shown in table 2 indicates a mean of 4.11 out of maximum obtainable score of 6 points, and standard deviation of 2.27. This standard deviation shows a low variability among the scores. Overall, there are twenty seven (45.8%) pre-service teachers who attained a score below the mean, whereas, thirty two (54.2%) were placed above the mean. Using the mean as a benchmark, it would be inferred that the pre-service teachers have performed remarkably well in this task. The qualitative data were generated using pre-service teachers' test manuscripts. Content analysis of their manuscripts indicated that some were able to define the basic trigonometric ratios clearly using the given variables, whereas, others use definition without using variables. Further analysis shows pre-service teachers were able to equate basic definitions trigonometric ratios with appropriate variables representing sides of right-angle triangle. There are various misconceptions, mistakes and confusions among those who performed below average. Frequency count indicated that 11(18.6%) have scored 0, while 16(27.1%) scored 2–4.5 points. Among low performers, some were able to recall and state three basic trigonometric ratios, but were unable to give the definitions. Another mistake was that basic ratios where given in terms of sides of right angle triangle with reference to the given angle $(\alpha)$, but associating those appropriate sides with the given variables was difficult or even impossible. Contrary, there are those who successfully associated the sides of the triangle with appropriate given variables (diagram) but unable to use them to define the trigonometric ratios. Lastly, there are those who clearly didn't attempt to answer the question. Similarly, there are those who are totally confused on what to do, they instead stated different trigonometric identities. Most common is $sin^2x + cos^2x = 1$. Summarizing the outcome, a fairly sizeable percentage exhibited a good conceptual understanding [20] of basic definition of triangle trigonometric ratios. Notwithstanding this success story, others were precluded in achieving this feat because of lack of good conceptual understanding.

### 6.2. Research question 2:

Are the pre-service teachers able to solve right-angle triangle problems?

Three variables i.e. ABKTR (apply basic knowledge of trigonometric ratio), VRTP (visualize right triangle trigonometric problem) and SRTP (solve right triangle trigonometric problem) were used to measure this research question. Previous experience of the pre-service teachers have positioned them in a situation that would enable them to calculate unknown specified side(s) of regular right angle triangles (ABKTR), visualize and sketch word problems leading to a right angle triangle (VRTP) as well as obtaining its solution (SRTP) confidently and fluently. Quantitative results from descriptive statistics show the mean of 4.31 and standard deviation of 6.01, and 20 points maximum obtainable for ABKTR; mean 2.20 and standard deviation of 1.72 with 5 points maximum obtainable for VRTP. Similarly, SRTP has a mean of 0.70 and standard deviation of 1.48, 5 points maximum obtainable. Assessing these statistical results, it can be concluded that the pre-service teachers' performance was very poor and dismal. A frequency count on the performance of the pre-service teachers for further qualitative analysis shows more revealing and supporting evidence. In the first variable ABKTR, 26(44.1%) of them scores 0 point, 11(18.6%) had scores ranging from 0.5-4.5 points, whereas 15(25.5%) obtained a score that varies between 5-6 points. Equally, 7(11.9%) achieved a score varying between $12-20$ points.

ABKTR requires direct application of trigonometric ratios (sine, cosine and tangent) from the given diagrams. Unfortunately, establishing a relationship between the given angle(s), side(s), and the unknown side(s) required to be calculated using appropriate trigonometric ratios was unattainable. Additionally, they could not split an unknown side represented by a variable into two sub-variables. This inability has hindered the computational procedures and limits their possibility of getting the correct solution. This is a clear demonstration of lack of procedural fluency [20]. Visualization ability was the hallmark of the variable VRTP. Pre-service teachers were required to sketch a diagram, specifically, right angle triangle representing the information given in a word problem. Frequency counts shows that $15(25.4\%)$ obtained a score of $0$, $28(47.5\%)$ earned between 1-2.5 points, while $16(27.1\%)$ scores 3-5 points. They were unable to visualize and sketch a right angle triangle, label it and use an appropriate trigonometric ratio to calculate and solve the problem. They were unable to locate the angle of elevation as well as the distance from a fixed point to the base of the mast. The last variable measuring this research question is SRTP. It requires setting up the problem and solves it using appropriate trigonometric ratios. Descriptive statistics indicate that $45(76.3\%)$ obtained a score of $0, 8(13.6\%)$ scores between 1–2.5 points, similarly, $6(10.2\%)$ earned a score that ranges between $3 - 5$ points. Since they were unable to visualize and sketch an appropriate diagram representing the word problem, it was not a surprise that they performed poorly in this task. Qualitative analysis shows that setting the problem was difficult among the few that were able to visualize and draw the diagram correctly. Moreover, there are cases were the choice of appropriate trigonometric ratio required to solve the problem was challenging. Similarly, finding the trigonometric value of $\tan 60^o$ was equally strenuous. To further support the short comings of the pre-service teachers, some made the following remarks: "I can't answer this question because I didn't understand it ... it is lack of practicing secondary school work; I can't recall the special angle values because they are difficult". Other comments that depict their deficiency in visualization ability and lack of effort to make connections between right-angle triangle and the trigonometric ratios are: "I forget the formula for finding the height of aerial mast; I can't remember because I have already forgotten it; I have to look back to my secondary school note and read over". Summarizing the signposting for this research question, the pre-service teachers were unable to create a link between the given angle(s), side(s) and unknown side(s) required to be calculated; they were also unable to visualize, sketch and label diagrams using appropriate variables as well as use suitable trigonometric ratios. Equally, finding trigonometric values of special angles was proving impossible. In effect, it could be inferred that the pre-service teachers were incapable of demonstrating procedural fluency and strategic competence [20].

### 6.3. Research question 3: Are the pre-service teachers able to recall and state sine and cosine rule ?

One variable BKNRT (Basic knowledge of non-right angle triangle) was used to measure this research question. The task requires pre-service teachers to recall and state formulas that were required to solve problems involving non-right angle triangles. The given problem dictates which formula (sine rule or cosine rule) would be used to solve it. Quantitatively, as contained in table 1, the descriptive statistics shows a mean score of $1.33$, out of $4$ maximum points obtainable, and deviation of $1.53$. The low standard deviation indicates that the variability among the pre-service teachers in this variable is minimal. To further support and consolidate this position, frequency tally shows $28(47.5\%)$ attained a score of $0, 19(32.2\%)$ earned a score ranging from $0.5$ to $3$, whereas, $12(20.3\%)$ gain a score of $3.50$ to $4$ points. Qualitative analysis of the pre-service teachers' manuscripts shows that many could not recall and state the sine and cosine rule/formula. However, few were able to state names of the formulas without stating the actual formulas. There are also some who end-off

writing area formula of triangle, example, A=1/2 ab; whereas, others state the Pythagorean Theorem formula, i.e. $a^2 = b^2 + c^2$ to represent sine/cosine formula. Clearly, it would be summarized that most of the pre-service teachers have a weak conceptual understanding of the sine and cosine rule/formula. According to Adding it up, good "conceptual understanding support retention and if forgotten can be reconstructed". There is no evidence from the pre-service teachers' manuscript demonstrating an attempt to reconstruct sine/cosine formula.

### 6.4. Research question 4 : What is the pre-service teachers' ability in solving non-right angle triangle problem ?

The last sets of variables measuring the final research question are VNRTP (visualize non-right angle triangle problem) and SNRTP (solve non-right angle triangle problem). They assess pre-service teachers' ability to visualize, sketch/draw diagrams representing non-right angle triangle problems, as well as formulate and solve the problems. Descriptive statistics from table 1 gives a mean of 2.41 and standard deviation of 2.08 for the variable VNRTP, while 0 was the mean and standard deviation for the other variable SNRTP. Maximum score for both variables is 5 point. Further statistical information also shows that 18(30.5%) obtained score of $0, 17(28.8\%)$ earned 0.5 to 2.5 points, while, 24(40.7%) get a score between $3 - 5$ points. Qualitative analysis of the manuscripts shows that the pre-service teachers were unable to visualize and draw diagrams representing a problem whose final structure denoted a non-right angle triangle. Similarly, they were also unable to put additional information, i.e. labelling the resultant diagram using appropriate variables. In the third component of the last research question i.e. the variable, SNRTP, the pre-service teachers' performance was dismal. In fact, none of the pre-service teachers were able to formulate mathematical equations using suitable sine or cosine rules to solve the problem. Qualitative checks of their manuscripts illustrate further their shortcomings as claimed above: "I can't answer because I have already forgotten ... need to look back at my secondary note book". Additionally, some asserted that "I have forgotten the way of solving it"; "I couldn't remember the formula under bearing". "I can't recall the formula exactly and which of sine or cosine formula to use". At this juncture, we could extrapolate that the pre-service teachers have undoubtedly display nonexistence of strategic competence, [20].

## 7. Conclusion, Discussion and Recommendation

Generally, the intent of this research is to appraise the mathematical proficiency of pre-service teachers in triangle trigonometry. Triangle trigonometry is a significant component of the junior and senior secondary school mathematics curriculum in Nigeria. Quantitative and qualitative data were generated and analyzed, uncovering the extent of the pre-service teachers' mathematical proficiency in triangle trigonometry or otherwise. A fragile grasp of conceptual understanding of basic trigonometric ratios, sine and cosine rules in isolated and non-isolated situations was exhibited by the pre-service teachers, even though few have shown an extremely poor comprehension. This finding is consistent with [12] as well as [1]. Isolated conceptual cases are where they were required to recall/state basic trigonometric ratios as well as sine and cosine rules; whereas, in non-isolated conceptual situations, the pre-service teachers are oblige to apply those basic knowledge where necessary. A good background in conceptual understanding supports retention [20] as well as ability to reconstruct forgotten generalization/formulas. Qualitative analysis of the pre-service teachers' manuscripts indicates that there was no evidence that the pre-service teachers have attempted to reconstruct basic trigonometric ratios, sine and cosine rule. Rather, some of them made comments such as "it was a secondary school material taken long ago". The solution of non-isolated cases was thwarted by the pre-service teachers' inability to visualize, sketch and

label right and non-right angle triangle problems and apply appropriate basic trigonometric ratios, sine and cosine rules to solve the given word problem. In similar vein, this outcome is coherent with previous ones, for example, [31]; [12]. In fact, as cited [32];[17]] results were even more appealing to the current study. It was stated that "Kendal and Stacey gave 178 high school students a trigonometry test one year after they completed a unit in which they studied trigonometry; 172 of these students scored a zero on this test". This tendency indicates that pre-service teachers' conceptual understanding, procedural fluency and strategic competence [20] are weak. The consequence of these dismal performances was that they were unable to drive mathematical equation and solve it from the resultant right and non-right angle diagrams in the few fairly successful cases. Based on the aforementioned outcomes, it is highly recommended as a matter of priority and urgency for the teacher education policy makers as well as teacher education institutions in Nigeria to incorporate teaching of junior and senior secondary school mathematics (trigonometric) content in the regular teacher education program.

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