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NANO $\widetilde{G}\alpha$-CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract: The basic objective of this paper is to introduce and investigate the properties of Nano $\widetilde{g}\alpha$-closed sets in Nano topological spaces.

Keywords. $N\widetilde{g}\alpha$-closed set, $N\widehat{g}$-closed set, N^*g-closed set and $N^\sharp gs$-closed set.
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1. Introduction

It was N. Levine [4] who introduced the concept of generalized closed sets in 1970. The concept of $\widetilde{g}\alpha$-closed sets was introduced by R. Devi et al. [2]. In 2013, M. Lellis Thivagar [3] has introduced the important and useful notion of Nano topological space with respect to the subset Ξ of a universe $\mho$. This type of topology is defined by utilizing the notions of approximations and boundary region of a universe by referring to an equivalence relation on it. Moreover, he also offered the new and interesting notions of Nano closed sets, Nano-interior and Nano-closure of a set. He has also introduced, among others, some certain weak forms of Nano open sets such as Nano α-open sets, Nano semi-open sets and Nano pre-open sets. The aim of this paper is to introduce a new class of sets on Nano topological spaces called Nano $\widetilde{g}\alpha$-closed sets. Further, we investigate and discuss the relation of this new sets with existing ones.

2. Preliminaries

Definition 2.1 [3] Let $\mho \neq \emptyset$ be a finite set of objects called the universe and $\Re$ be an equivalence relation. The elements which belong to the same equivalence class are said to be indiscernible with one another. The pair $(\mho, \Re)$ is said to be the approximation space. Let $\Xi \subseteq \mho$, then

(i) The lower approximation of Ξ w.r.t. $\Re$ is the set of all objects, which can be for certain classified as Ξ w.r.t. $\Re$ and its denoted by $L_\Re(\Xi)$ and $L_\Re(\Xi) = \bigcup\limits_{x\in\mho} \{\Re(x) : (x) \subseteq \Xi\}$, where $\Re(x)$ denotes the equivalence class determined by $x \in \mho$.

(ii) The upper approximation of Ξ w.r.t. $\Re$ is the set of all objects, which can be possibly classified as Ξ w.r.t. $\Re$ and it is denoted by $\mho_{(\Xi)}$ which $\mho_\Re(\Xi) = \bigcup\limits_{x\in\mho} \{\Re(x) : \Re(x) \cap \Xi \neq \phi\}$.

(iii) The boundary region of Ξ w.r.t. $\Re$ is the set of all objects which can be classified neither as Ξ nor as not Ξ with respect to $\Re$ and it is denoted by $B_\Re(\Xi)$ and $B_\Re(\Xi) = \mho_\Re(\Xi) - L_\Re(\Xi)$.

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Property 2.2. [3] If $(\mho, \Re)$ is an approximation space and $\Xi, \Lambda \subseteq \mho$, then

(i) $L_{\Re}(\Xi) \subseteq \Xi \subseteq \mho_{\Re}(\Xi)$.

(ii) $L_{\Re}(\phi) = \mho_{\Re}(\phi) = \phi$ and $L_{\Re}(\mho) = \mho_{\Re}(\mho) = \mho$.

(iii) $\mho_{\Re}(\Xi \cup \Lambda) = \mho_{\Re}(\Xi) \cup \mho_{\Re}(\Lambda)$.

(iv) $L_{\Re}(\Xi \cup \Lambda) \supseteq L_{\Re}(\Xi) \cup L_{\Re}(\Lambda)$.

(v) $\mho_{\Re}(\Xi \cap \Lambda) \subseteq \mho_{\Re}(\Xi) \cap \mho_{<} Re(\Lambda)$.

(vi) $L_{\Re}(\Xi \cap \Lambda) = L_{\Re}(\Xi) \cap L_{\Re}(\Lambda)$.

(vii) $L_{\Re}(\Xi) \subseteq L_{\Re}(\Lambda)$ and $\mho_{\Re}(\Xi) \subseteq \mho_{\Re}(\Lambda)$, whenever $\Xi \subseteq \Lambda$.

(viii) $\mho_{\Re}(\Xi^c) = [L_{\Re}(\Xi)]^c$ and $L_{\Re}(\Xi^c) = [\mho_{\Re}(\Xi)]^c$.

(ix) $\mho_{\Re}\mho_{\Re}(\Xi) = L_{\Re}\mho_{\Re}(\Xi) = \mho_{\Re}(\Xi)$.

(x) $L_{\Re}L_{\Re}(\Xi) = \mho_{\Re}L_{\Re}(\Xi) = L_{\Re}(\Xi)$.

Definition 2.3. [3] Let $\mho$ be the universe, $\Re$ be an equivalence relation on $\mho$ and $\tau_{\Re}(\Xi) = \{\mho, \phi, L_{\Re}(\Xi), \mho_{\Re}(\Xi), B_{\Re}(\Xi)\}$, where $\Xi \subseteq \mho$. Then by property 2.2, $\tau_{\Re}(\Xi)$ satisfies the following axioms:

(i) $\mho$ and $\phi \in \tau_{\Re}(\Xi)$.

(ii) The union of the elements of any subcollection of $\tau_{\Re}(\Xi)$ is in $\tau_{\Re}(\Xi)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{\Re}(\Xi)$ is in $\tau_{\Re}(\Xi)$.

$\tau_{\Re}(\Xi)$ is a topology on $\mho$ called the Nano topology on $\mho$ with respect to Ξ. We denote the Nano topological space by $(\mho, \tau_{\Re}(\Xi))$ as . The elements of $\tau_{\Re}(\Xi)$ are called Nano open sets.

Remark 2.4. [3] If $\tau_{\Re}(\Xi)$ is the Nano topology on $\mho$ with respect to Ξ, the the set $B = \{\mho, L_{\Re}(\Xi), \mho_{\Re}(\Xi)\}$ is the basis for $\tau_{\Re}(\Xi)$.

Definition 2.5. [3] If $(\mho, \tau_{\Re}(\Xi))$ is a Nano topological space with respect to Ξ, where $\Xi \subseteq \mho$ and if $A \subseteq \mho$, then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest Nano open subset of A. The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A.

Definition 2.6. [3] A Nano topological space $(\mho, \tau_{\Re}(\Xi))$ is said to be extremely disconnected if the Nano closure of each Nano open set is Nano open.

Definition 2.7. [3] Let $(\mho, \tau_{\Re}(\Xi))$ be a Nano topological space and $A \subseteq \mho$. Then A is said to be

(i) Nano semi open if $A \subseteq Ncl(Nint(A))$

(ii) Nano α-open if $A \subseteq Nint(Ncl(Nint(A)))$

The Nano α-closure of A is defined as the intersection of all Nano α-closed sets containing A and it is denoted by $N\alpha cl(A)$. This means that $N\alpha cl(A)$ is the smallest Nano α-closed set containing A. The Nano semi-closure of A is defined as the intersection of all Nano semi-closed sets containing A and it is denoted by $Nscl(A)$. $Nscl(A)$ is the smallest Nano semi closed set containing A. We recall the following definitions which are useful in the sequel.

Definition 2.8. Let $\mho$ be a Nano topological space. A subset A of $\mho$ is called

(a) Nano semi-generalized closed (briefly Nsg-closed) set [1] if $Nscl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is Nano semi-open in $\mho$,

(b) Nano generalized semi-closed (briefly Ngs-closed) set [1] if $Nscl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is Nano open in $\mho$,

(c) Nano α-generalized closed (briefly $N\alpha g$-closed) set [5] if $N\alpha cl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is Nano open in $\mho$,

(d) Nano generalized α-closed (briefly $Ng\alpha$-closed) set [5] if $N\alpha cl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and V is $N\alpha$-open in $\mho$.

3. Properties of Nano $\widetilde{g}\alpha$-closed sets

We introduce the following definitions.

Definition 3.1. Let $\mho$ be a Nano topological space. A subset A of $\mho$ is called

(a) Nano $\widehat{g}$-closed set (briefly $N\widehat{g}$-closed) if $Ncl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is Nano semi-open in $\mho$,

(b) Nano ${}^*g$-closed set (briefly N^*g-closed) if $Ncl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is $N\widehat{g}$-open in $\mho$,

(c) Nano ${}^\sharp gs$-closed set (briefly $N^\sharp gs$-closed) if $Nscl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is N^*g-open in $\mho$.

In this section, we define and study the forms of Nano $\tilde{g}\alpha$-closed sets.

Definition 3.2. Let $\mho$ be a Nano topological space. A subset A of $\mho$ is called Nano $\widetilde{g}\alpha$-closed set (briefly $N\widetilde{g}\alpha$-closed) if $N\alpha cl(A) \subseteq \Upsilon$, whenever $A \subseteq \Upsilon$ and Υ is $N^\sharp gs$-open in $\mho$.

Theorem 3.3.

(a) Every $N\alpha$-closed set is a $N\widetilde{g}\alpha$-closed set.

(b) Every $N\widetilde{g}\alpha$-closed set is a Ngs-closed set.

(c) Every $N\widetilde{g}\alpha$-closed set is a $Ng\alpha$-closed set.

(d) Every $N\widetilde{g}\alpha$-closed set is a $N\alpha g$-closed set.

(e) Every $N\widetilde{g}\alpha$-closed set is a Nsg-closed set.

Proof.

(a) Let A be an $N\alpha$-closed set in $\mho$, then $A = N\alpha cl(A)$. Let $A \subseteq \Upsilon$, Υ is $N^{\sharp}gs$-open in $\mho$. Since A is $N\alpha$-closed, $A = \alpha cl(A) \subseteq \Upsilon$. This shows that A is $N\widetilde{g}\alpha$-closed set.

(b) Let A be an $N\widetilde{g}\alpha$-closed set in $\mho$. Let $A \subseteq \Upsilon$, Υ is a nano open in $\mho$ which implies Υ is an $N^{\sharp}gs$-open set. Since A is $N\widetilde{g}\alpha$-closed, $Nscl(A) \subseteq N\alpha cl(A) \subseteq \Upsilon$. This shows that A is Ngs-closed set.

(c) Let A be an $N\widetilde{g}\alpha$-closed set in $\mho$. Let $A \subseteq \Upsilon$, Υ is an $N\alpha$-open set in $\mho$ which implies Υ is an $N^{\sharp}gs$-open set. Since A is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq \Upsilon$. This shows that A is an $Ng\alpha$-closed set.

(d) Let A be an $N\widetilde{g}\alpha$-closed set in $\mho$. Let $A \subseteq \Upsilon$, Υ is a Nano open in U which implies Υ is an $N^{\sharp}gs$-open set. Since A is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq \Upsilon$. This shows that A is an $N\alpha g$-closed set.

(e) Let A be an $N\widetilde{g}\alpha$-closed set in $\mho$. Let $A \subseteq \Upsilon$, Υ is a nano semi open in $\mho$ which implies Υ is an $N^{\sharp}gs$-open set. Since A is $N\widetilde{g}\alpha$-closed, $Nscl(A) \subseteq N\alpha cl(A) \subseteq \Upsilon$. This shows that A is an Nsg-closed set.

The following examples show that these implications are not reversible.

Example 3.4.

(a) Let $\mho = \{a, b, c, d\}$ with $\mho/\Re = \{\{a\}, \{c\}, \{b, d\}\}$ and $\Xi = \{a, b\}$. Thus the Nano topology, $\tau_{\Re}(\Xi) = \{\mho, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Then in a space $\mho$, a subset $\{a, b, c\}$ is an $N\widetilde{g}\alpha$-closed set but it is not an $N\alpha$-closed set.

(b) Let $\mho = \{a, b, c, d\}$ with $\mho/\Re = \{\{a\}, \{c\}, \{b, d\}\}$ and $\Xi = \{a, b\}$. Then the Nano topology, $\tau_{\Re}(\Xi) = \{\mho, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. Hence in a space $\mho$, a subset $\{a\}$ is an Ngs-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(c) Let $\mho = \{a, b, c, d\}$ with $\mho/\Re = \{\{a\}, \{c\}, \{b, d\}\}$ and $\Xi = \{a, b, c\}$. Then the Nano topology, $\tau_{\Re}(\Xi) = \{\mho, \phi, \{a, c\}, \{b, d\}\}$. In a space $\mho$, a subset $\{a, b, c\}$ is an $Ng\alpha$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(d) Let $\mho = \{a, b, c, d, e\}$ with $\mho/\Re = \{\{d\}, \{a, b\}, \{c, e\}\}$ and $\Xi = \{a, d\}$. Then the Nano topology, $\tau_{\Re}(\Xi) = \{\mho, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$. Then in a space $\mho$, a subset $\{a, c\}$ is an $N\alpha g$-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

(e) Let $\mho = \{a, b, c, d\}$ with $\mho/\Re = \{\{a\}, \{c\}, \{b, d\}\}$ and $\Xi = \{a, b\}$. Then the Nano topology, $\tau_{\Re}(\Xi) = \{\mho, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$. In a space $\mho$, a subset $\{b\}$ is an Nsg-closed set but it is not an $N\widetilde{g}\alpha$-closed set.

Theorem 3.5. Let A be a subset of a Nano topological space $\mho$. If A is $N\widetilde{g}\alpha$-closed, then $N\alpha cl(A) - A$ does not contain any non-empty $N^{\sharp}gs$-closed set.

Proof. Suppose that A is $N\widetilde{g}\alpha$-closed and let Φ be a non-empty $N^{\sharp}gs$-closed set with $\Phi \subseteq N\alpha cl(A) - A$. Then $A \subseteq \mho - \Phi$ and so $N\alpha cl(A) \subseteq \mho - \Phi$. Hence $\Phi \subseteq \mho - N\alpha cl(A)$ which is a contradiction.

Theorem 3.6. Let A be a subset of a Nano topological space $\mho$. If A is $N\widetilde{g}\alpha$-closed and $A \subseteq B \subseteq N\alpha cl(A)$, then B is $N\widetilde{g}\alpha$-closed.

Proof. Let Υ be an $N^{\sharp}gs$-open set of $\mho$ such that $B \subseteq \Upsilon$. Then $A \subseteq \Upsilon$. Since A is $N\widetilde{g}\alpha$-closed, then

$N\alpha cl(A) \subseteq \Upsilon$. Now $N\alpha cl(B) \subseteq N\alpha cl(N\alpha cl(A)) \subseteq \Upsilon$. Therefore B is also an $N\widetilde{g}\alpha$-closed set of $\mho$.

Theorem 3.7. Let A and B be $N\widetilde{g}\alpha$-closed sets of a Nano topological space $\mho$. Then $A \cup B$ is an $N\widetilde{g}\alpha$-closed set in $\mho$.

Proof. Let A and B be $N\widetilde{g}\alpha$-closed sets. Let $A \cup B \subseteq \Upsilon$, where Υ is $N^\sharp gs$-open. Since A and B are $N\widetilde{g}\alpha$-closed sets, $N\alpha cl(A) \subseteq \Upsilon$ and $N\alpha cl(B) \subseteq \Upsilon$. This implies that $N\alpha cl(A \cup B) = N\alpha cl(A) \cup N\alpha cl(B) \subseteq \Upsilon$ and so $N\alpha cl(A \cup B) \subseteq \Upsilon$. Therefore $A \cup B$ is $N\widetilde{g}\alpha$-closed.

Lemma 3.8. For any space $\mho$, $\mho = \mho_{N^\sharp gsc} \cup \mho_{N\widetilde{g}\alpha o}$ holds.

Proof. Let $x \in \mho$. Suppose that $\{x\}$ is not $N^\sharp gs$-closed set in $\mho$. Then $\mho$ is a unique $N^\sharp gs$-open set containing $\mho - \{x\}$. Thus $\mho - \{x\}$ is $N\widetilde{g}\alpha$-closed in $\mho$ and so $\{x\}$ is $N\widetilde{g}\alpha$-open. Therefore $x \in \mho_{N^\sharp gsc} \cup \mho_{N\widetilde{g}\alpha o}$ holds.

We need more notations:

For a subset A of $\mho$, $ker(A) = \cap\{\Upsilon / \Upsilon \in \tau_{\Re}(\Xi)$ and $A \subseteq \Upsilon\}$;
$N^\sharp GSO\text{-}ker(A) = \cap\{\Upsilon/\Upsilon \in N^\sharp GSO(\mho)$ and $A \subseteq \Upsilon\}$.

Theorem 3.9. For a subset A of $\mho$, the following conditions are equivalent.

(1) A is $N\widetilde{g}\alpha$-closed in $\mho$.

(2) $N\alpha cl(A) \subseteq N^\sharp GSO\text{-}ker(A)$ holds.

(3) (i) $N\alpha cl(A) \cap \mho_{N^\sharp gsc} \subseteq A$ and (ii) $N\alpha cl(A) \cap \mho_{N^\sharp gso} \subseteq N^\sharp GSO\text{-}ker(A)$ holds.

Proof.
(1) $\Rightarrow$ (2) Let $x \notin N^\sharp GSO\text{-}ker(A)$. Then there exists a set $\Upsilon \in N^\sharp GSO(\mho)$ such that $x \notin \Upsilon$ and $A \subseteq \Upsilon$. Since A is $N\widetilde{g}\alpha$-closed, $N\alpha cl(A) \subseteq \Upsilon$ and so $x \notin N\alpha cl(A)$. This shows that $N\alpha cl(A) \subseteq N^\sharp GSO\text{-}ker(A)$.
(2) $\Rightarrow$ (1) Let $\Upsilon \in N^\sharp GSO(\mho)$ such that $A \subseteq \Upsilon$. Then, we have that $N^\sharp GSO\text{-}ker(A) \subseteq \Upsilon$ and so by (2) $N\alpha cl(A) \subseteq \Upsilon$. Therefore A is $N\widetilde{g}\alpha$-closed.
(2) $\Rightarrow$ (3) (i) First we claim that $N^\sharp GSO\text{-}ker(A) \cap \mho_{N^\sharp gsc} \subseteq A$. Indeed, let $x \in N^\sharp GSO\text{-}ker(A) \cap \mho_{N^\sharp gsc}$ and assume that $x \notin A$. Since the set $\mho - \{x\} \in N^\sharp GSO(\mho)$ and $A \subseteq \mho - \{x\}$, $N^\sharp GSO\text{-}ker(A) \subseteq \mho - \{x\}$. Then we have that $x \in \mho - \{x\}$ and so this is a contradiction. Thus we show that $N^\sharp GSO\text{-}ker(A) \cap \mho_{N^\sharp gsc} \subseteq A$. By using (2), $N\alpha cl(A) \cap \mho_{N^\sharp gsc} \subseteq N^\sharp GSO\text{-}ker(A) \cap \mho_{N^\sharp gsc} \subseteq A$.
(ii) It is obtained by (2).
(3) $\Rightarrow$ (2) By Lemma 3.8 and (3),

$$\begin{aligned} N\alpha cl(A) &= N\alpha cl(A) \cap \mho = N\alpha cl(A) \cap (\mho_{N^\sharp gsc} \cup \mho_{N\widetilde{g}\alpha o}) \\ &= (N\alpha cl(A) \cap \mho_{N^\sharp gsc}) \cup (N\alpha cl(A) \cap \mho_{N\widetilde{g}\alpha o}) \\ &= A \cup N^\sharp GSO\text{-}ker(A) \\ &= N^\sharp GSO\text{-}ker(A) \text{ holds.} \end{aligned}$$

Theorem 3.10. Let $\mho$ be a Nano topological space and $A \subseteq \mho$.

(a) If A is $N^\sharp gs$-open and $N\widetilde{g}\alpha$-closed, then A is α-closed in $\mho$.

(b) Suppose that $\mho$ is an $N\alpha$-space. An $N\widetilde{g}\alpha$-closed set A is $N\alpha$-closed in $\mho$ if and only if $N\alpha cl(A) - A$ is $N\alpha$-closed in $\mho$.

(c) For each $x \in \mho$, $\{x\}$ is $N^\sharp gs$-closed or $\mho - \{x\}$ is $N\widetilde{g}\alpha$-closed in $\mho$.

(d) Every subset is $N\widetilde{g}\alpha$-closed in $\mho$ if and only if $N^\sharp gs$-open set is $N\alpha$-closed.

Proof.

(b) (Necessity) If A is $N\alpha$-closed, then $N\alpha cl(A) - A = \phi$.
(Sufficiency) Suppose that A is $N\widetilde{g}\alpha$-closed and $N\alpha cl(A) - A$ is $N\alpha$-closed. Then, $N\alpha cl(A) - A$ is $N^\sharp gs$-closed in $\mho$ and by Theorem 3.5, $N\alpha cl(A) - A = \phi$. Therefore A is $N\alpha$-closed in $\mho$.

(c) If $\{x\}$ is not $N^\sharp gs$-closed, then $\mho - \{x\}$ is not $N^\sharp gs$-open. Therefore $\mho - \{x\}$ is $N\widetilde{g}\alpha$-closed in $\mho$.

(d) (Necessity) Let Υ be an $N^\sharp gs$-open set. Then we have that $N\alpha cl(\Upsilon) \subseteq \Upsilon$ and hence Υ is $N\alpha$-closed.
(Sufficiency) Let A be a subset and Υ is an $N^\sharp gs$-open set such that $A \subseteq \Upsilon$. Then $N\alpha cl(A) \subseteq N\alpha cl(\Upsilon) = \Upsilon$ and hence A is $N\widetilde{g}\alpha$-closed.

4. Properties of Nano $\widetilde{g}\alpha$-open sets

Definition 4.1. A subset A of a Nano topological space $\mho$ is called Nano $\widetilde{g}\alpha$-open set (briefly $N\widetilde{g}\alpha$-open) if A^c is $N\widetilde{g}\alpha$-closed.

Theorem 4.2. A subset $A \subseteq \mho$ is $N\widetilde{g}\alpha$-open if and only if $\Phi \subseteq N\alpha int(A)$ whenever Φ is $N^\sharp gs$-closed set and $\Phi \subseteq A$.

Proof. Let A be an $N\widetilde{g}\alpha$-open set and suppose $\Phi \subseteq A$, where Φ is $N^\sharp gs$-closed. Then $\mho - A$ is $N\widetilde{g}\alpha$-closed set contained in $N^\sharp gs$-open set $\mho - \Phi$. Hence $N\alpha cl(\mho - A) \subseteq \mho - \Phi$ and $\mho - N\alpha int(A) \subseteq \mho - \Phi$. Thus $\Phi \subseteq N\alpha int(A)$.

Conversely, if Φ is $N^\sharp gs$-closed set with $\Phi \subseteq N\alpha int(A)$ and $\Phi \subseteq A$, then $\mho - N\alpha int(A) \subseteq \mho - \Phi$. Thus $N\alpha cl(\mho - A) \subseteq \mho - \Phi$. Hence $\mho - A$ is an $N\widetilde{g}\alpha$-closed set and A is an $N\widetilde{g}\alpha$-open set.

Theorem 4.3. Let A and B be subsets of a Nano topological space $\mho$. If $N\alpha int(A) \subseteq B \subseteq A$ and A is $N\widetilde{g}\alpha$-open, then B is $N\widetilde{g}\alpha$-open.

Proof. Let $N\alpha int(A) \subseteq B \subseteq A$. Then $A^c \subseteq B^c \subseteq N\alpha cl(A^c)$, where A^c is $N\widetilde{g}\alpha$-closed and hence B^c is also $N\widetilde{g}\alpha$-closed by Theorem 3.6. Therefore, B is $N\widetilde{g}\alpha$-open.

Theorem 4.4. Let A be a subset of a Nano topological space $\mho$. If A is $N\widetilde{g}\alpha$-closed, then $N\alpha cl(A) - A$ is $N\widetilde{g}\alpha$-open.

Proof. Let A be $N\widetilde{g}\alpha$-closed and Φ be $N^\sharp gs$-closed such that $\Phi \subseteq N\alpha cl(A) - A$. Then $\Phi = \phi$ by Theorem 3.5. Therefore $\Phi \subseteq N\alpha int(N\alpha cl(A) - A)$. Hence $N\alpha cl(A) - A$ is $N\widetilde{g}\alpha$-open.

Definition 4.5. Let $\mho$ be a Nano topological space and $x \in \mho$. A subset N of $\mho$ is said to be $N\widetilde{g}\alpha$-neighbourhood of x if there exists an $N\widetilde{g}\alpha$-open set G such that $x \in G \subseteq N$.

Definition 4.6.

(a) $N\widetilde{g}\alpha int(A) = \bigcup \Big\{ B : B \text{ is } N\widetilde{g}\alpha\text{-open set and } B \subseteq A \Big\}$.

(b) $N\widetilde{g}\alpha cl(A) = \bigcap \Big\{ B : B \text{ is } N\widetilde{g}\alpha\text{-closed set and } A \subseteq B \Big\}$.

Theorem 4.7. Let A and B be subsets of $\mho$. Then

(a) $N\widetilde{g}\alpha int(\mho) = \mho$ and $N\widetilde{g}\alpha int(\phi) = \phi$.

(b) $N\widetilde{g}\alpha int(A) \subseteq A$.

(c) If B is any $N\widetilde{g}\alpha$-open set contained in A, then $B \subseteq N\widetilde{g}\alpha int(A)$.

(d) If $A \subseteq B$, then $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(B)$.

Proof.

(a) Since $\mho$ and ϕ are $N\widetilde{g}\alpha$-open sets, by definition $N\widetilde{g}\alpha int(\mho) = \bigcup \Big\{ B : B \text{ is } N\widetilde{g}\alpha\text{-open set and } B \subseteq \mho \Big\} = \mho$. Since ϕ is the only $N\widetilde{g}\alpha$-open set contained in ϕ, $N\widetilde{g}\alpha int(\phi) = \phi$.

(b) Let $x \in Ng\widetilde{\alpha} int(A) \Rightarrow x$ is a $N\widetilde{g}\alpha$ interior of $A \Rightarrow A$ is a $N\widetilde{g}\alpha$-neighbourhood of $x \Rightarrow x \in A$. Thus $N\widetilde{g}\alpha int(A) \subseteq A$.

(c) Let B be any $N\widetilde{g}\alpha$-open set such that $B \subseteq A$. Let $x \in B$. Since B is an $N\widetilde{g}\alpha$-open set contained in A, x is an $N\widetilde{g}\alpha$-interior point of A. That is B is an $N\widetilde{g}\alpha int(B)$. Hence $B \subseteq N\widetilde{g}\alpha int(A)$.

(d) Let A and B be subsets of $\mho$ such that $A \subseteq B$. Let $x \in N\widetilde{g}\alpha int(A)$. Then x is an $N\widetilde{g}\alpha$-interior point of A and so A is $N\widetilde{g}\alpha$-neighbourhood of x. This implies that $x \in N\widetilde{g}\alpha int(B)$. Hence $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(B)$.

Theorem 4.8. If a subset A of a nano topological space $\mho$ is $N\widetilde{g}\alpha$-open, then $N\widetilde{g}\alpha int(A) = A$.
Proof. Let A be an $N\widetilde{g}\alpha$-open subset of $\mho$. We know that $N\widetilde{g}\alpha int(A) \subseteq A$. Also A is an $N\widetilde{g}\alpha$-open set contained in A. By Theorem 4.7 (c), $A \subseteq N\widetilde{g}\alpha int(A)$. Hence $N\widetilde{g}\alpha int(A) = A$.

Theorem 4.9. If A and B are subsets of $\mho$, then $N\widetilde{g}\alpha int(A) \cup N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$.
Proof. We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Theorem 4.7 (d), $N\widetilde{g}\alpha int(A) \subseteq N\widetilde{g}\alpha int(A \cup B)$ and $N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$. This implies that $N\widetilde{g}\alpha int(A) \cup N\widetilde{g}\alpha int(B) \subseteq N\widetilde{g}\alpha int(A \cup B)$.

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