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Vertex graceful and difference labeling of some special graphs

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Abstract: In this expose for any graph G having vertices V and edges E. A Graph with vertex V and edge E is said to have difference labeling of G ,if for an injection f from V to the set of non-negative integer together with weight function f^* on E given by $f^*(uv) = |f(u) - f(v)|$ for every edge $uv \in E$. A graph with a difference labeling on it is called a labeled graph. Also a graph with distinct vertex labeled graph is called vertex graceful labeling. This theory provides a viewpoint that helps provide new insights in the discovery of Vertex graceful and Difference labeling on gear graph G_n, swasthick graph S_n, coconut tree graph CT_n, $\prod$ tree, Y tree.

Key words: Difference Labelling, Weight Decomposition, Graceful Graphs.

1. Introduction

All the way through this paper, by a graph we despicable a finite, undirected, simple graph $G = (V(G), E(G))$ with p vertices and q edges. Graph Labeling is an energetic region belongs to research in graph theory which has an accurate purpose in coding theory, communicate networks and graph putrefaction problems, for notation and terminology we refer to Bondy and Muthy[2]. J.A.Gallian[1] developed graph labeling which performs as a boundary stuck flanked by number theory and the structure of graphs. A difference labeling of a graph G is grasped by assigning distinct integer values to its vertices and then connecting with each edge uv the absolute difference of those values consigned to its end vertices. The perception of difference Labelings was familiarized by G.S.Bloom and S.Ruiz [3].There are different types of graceful labeling like odd graceful labeling, even graceful labeling and skolem - graceful labeling to various classes of graphs.

2. Preliminaries

2.1. Labeled Graph

Let $G(V, E)$ be a graph. A difference labeling of G is an inoculation of G is an injection f from v to the set of non-negative integer with common weight function f^* on E given by $f^*(uv) = |f(u) - f(v)|$ for every edge uv in G. A graph with a difference labeling defined on it is called Labeled graph.

2.2. Common - Weight Decomposition

A decomposition of labeled graph hooked on parts, each part enclosing the edge consuming a common- weight is called a common - weight decomposition.

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2.3. Wheel Graph

The wheel graph W_n is de?ned to be the juncture of $K_1 + C_n$. The vertex resultant to K_1 is known as the apex vertex and the vertices resultant to cycle are known as rim vertices whereas the edges resultant to cycle are known as rim edges.

$$V(W_n) = \{c, v_1, v_2, \ldots v_n\}$$

$$E(W_n = \{cv_i/1 < i < n\} \cup \{v_i v_{i+1}/1 < i < n-1\} \cup \{v_n v_1\}$$

2.4. Gear Graph

The gear graph G_n is attained from wheel W_n using sub dividing every one of its rim edge.

2.5. Helm Graph

The helm H_n is the graph attained from wheel W_n by assigning a pendant edge to each rim vertex.

2.6. Swasthick Graph S_n

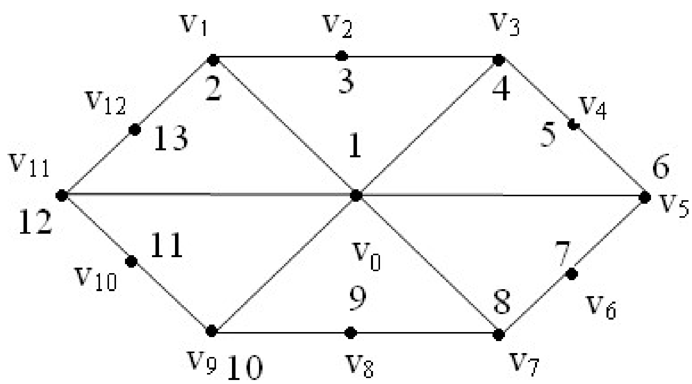
The Swasthick graph S_n is the graph attained from gear G_n by assigning a pendent edge to each vertex of the cycle of G_n

3. GENERAL RESULTS IN LABELED GRAPHS

Theorem 3.1. *The gear graph G_n is a Vertex Graceful and Difference Labeling Graph.*

Proof. Let $G = G_n$ be a gear graph.
Let $V(G) = \{v_0, v_1, v_2 \ldots v_{2n}\}$ and $E(G) = \{v_0 v_{2i-1}, 1 \leq i \leq n\} \cup \{v_i v_{i+1}, 1 \leq i \leq n-1\} \cup \{v_{2n} v_1\}$ with $|v(G)| = 2n+1$ and $E(G) = 3n$.
Define a function $f : V(G) \to \{1, 2, 3, \ldots 2n+1\}$ by
$f(v_0) = 1$
$f(v_i) = i+1, 1 \leq i \leq 2n$
Therefore every vertex has distinct values.
Define the weight function f^* on edge E by $f^*(v_i vi + 1) = |f(vi) - f(v_{i+1})|, 1 \leq i \leq 2n$.
Therefore f^* has the same value as 1 on every edge on the circumference.
Now we Define the weight function f^* on E for edge connecting the centre and the vertex on the circumference as $f^*(v_0 v_i) = |f(v_0) - f(v_i)|$
Therefore we have edge value f^* as $1, 3, 5, \ldots (2n-1)$.
Hence the gear graph G_n is a labeled graph.
Since all the vertices are distinct, G_n is Vertex Graceful and Difference Labeling Graph. □

Example 3.1. *Vertex Graceful and Difference Labeling Graph of G_6 is given below:*



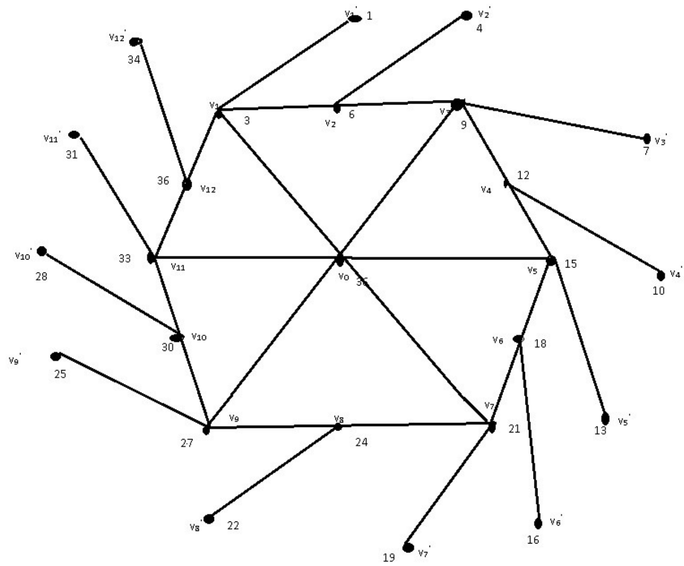
Theorem 3.2. *The Swasthick graph S_n is a Vertex Graceful Labeled Graph*

Proof. Let $G = S_n$ be a swasthick graph.
Let $V(G) = \{v_0, v_1, v_2 \ldots v_{2n}, v_1^{'}, v_2^{'} \ldots v_{2n}^{'}\}$ where v_0 be the centre vertex,

$v_i, 1 \leq i \leq n$ be the vertex on the circumference,
$v_i^{'}, 1 \leq i \leq n$ be the pendent vertex incident at each v_i

with $|v(G)| = 2n + 1$.
Let $E(G) = \{v_0 v_{2i-1}, 1 \leq i \leq 2n\} \cup \{v_{2i-1} v_{2i-1}', 1 \leq i \leq 2n\} \cup \{v_{2i} v_{2i}', 1 \leq i \leq 2n\} \cup \{v_i v_{i+1}, 1 \leq i \leq 2n\} \cup \{v_{2n} v_1\}$
With $|E(G)| = 3n$.
Now define a function $f : V(G) \to \{1, 2, 3, \ldots 4n + 1\}$ by $f(v_0) = 3 * 2n$
$f(v_i) = 3j, 1 \leq i \leq 2n$ and $0 \leq j \leq 2n - 1$
$f(v_i^{'}) = 3j - 2, 1 \leq i \leq 2n$ and $1 \leq j \leq 2n$
Therefore every vertex has distinct values.
Define the weight function f^* on edge E by $f^*(uv) = |f(u) - f(v)|$ then edge can be decomposed into different path values as 3 on every edge on the circumference and 2 on every edge of pendent vertex incident on circumference vertex and multiples of 3 on every edge on circumference to centre.
Hence the swasthick graph S_n is a labeled graph. Since all the vertices are distinct, S_n is Vertex Graceful and Difference Labeling Graph. □

Example 3.2. *Vertex Graceful and Difference Labeling Graph of swasthick graph S_6*

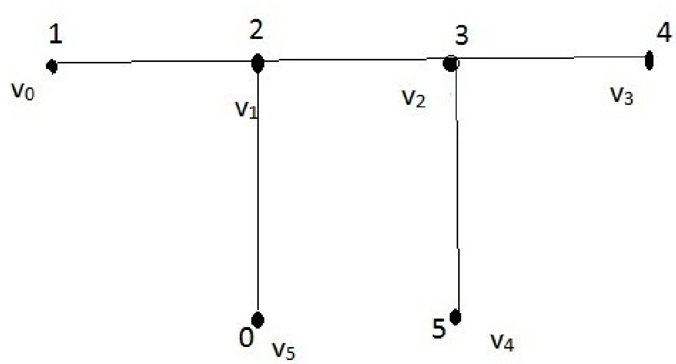


Theorem 3.3. *π-tree is a vertex graceful and Difference Labeling Graph.*

Proof. Let $V_0, V_1, \ldots\ldots V_{n-1}$ be the consequence vertices of the path $P_n(n \geq 4)$ the π-tree π_n is the tree of order $n+2$ whose vertex set is $V = \{v_0, v_{n-1}, v_n, v_{n+1}\}$ and edge set is $E = \{e_i = v_{i-1}v_i, 1 \leq i \leq n-1\} \cup \{e_n = v_{n-2}v_n, en+1 = v_1 v_{n+1}\}$.

(ie) π_n tree is obtained from path P_n by attaching two vertices v_n and v_{n+1} to the vertices v_{n-2} and v_1 respectively of P_n.
Now define the function f on vertices V as follows:
$f(v_i) = i + 1; 1 \leq i \leq n - 1$
$f(v_n) = n + 1$
$f(v_{n+1}) = 0$
Now we can define the edge value E as $f^*(uv) = |f(u) - f(v)|$.
Since all the vertices are distinct with two different sets of edges value1 and 5, π-tree is vertex graceful and Difference Labeling Graph. □

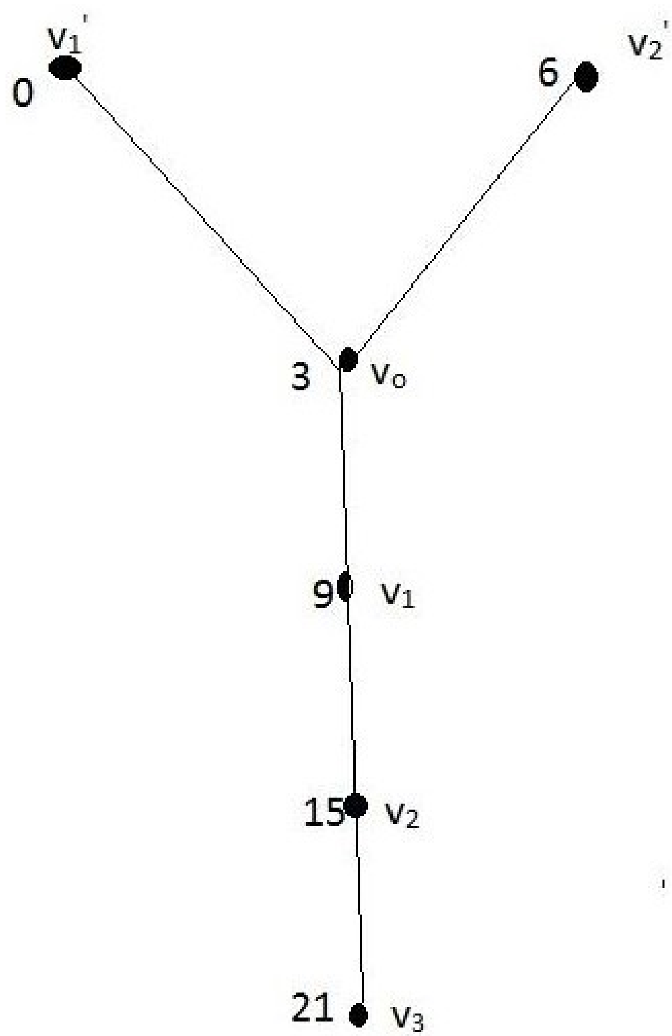
Example 3.3. *Vertex Graceful and Difference Labeling Graph of π_4*



Theorem 3.4. *Y - tree is a vertex graceful and Difference Labeling Graph.*

Proof. Let $V_0, V_1, \ldots\ldots Vn-1$ be the consecutive vertices of the path P_n.
The Y-tree Y_n is the tree of order $n+1$ whose vertex set is $V = \{v_0, v_1 \ldots v_n\} \cup \{v_1^{'}, v_2^{'}\}$ and edge set is $E = \{e_i = v_{i+1}vi, 0 \le i \le n-2\} \cup \{e' = v_1^{'}, v_2^{'}\}$.
Now define the function f on vertices V as follows: $f(v_i) = 6n-3; 0 \le i \le n-2$
Now we can define the edge value as $f^*(uv) = |f(u) - f(v)|$.
Since all the vertices are distinct with two different sets of edges value, π-tree is vertex graceful labelled graph. □

Example 3.4. *Vertex Graceful and Difference Labeling Graph of Y_6 Graph*

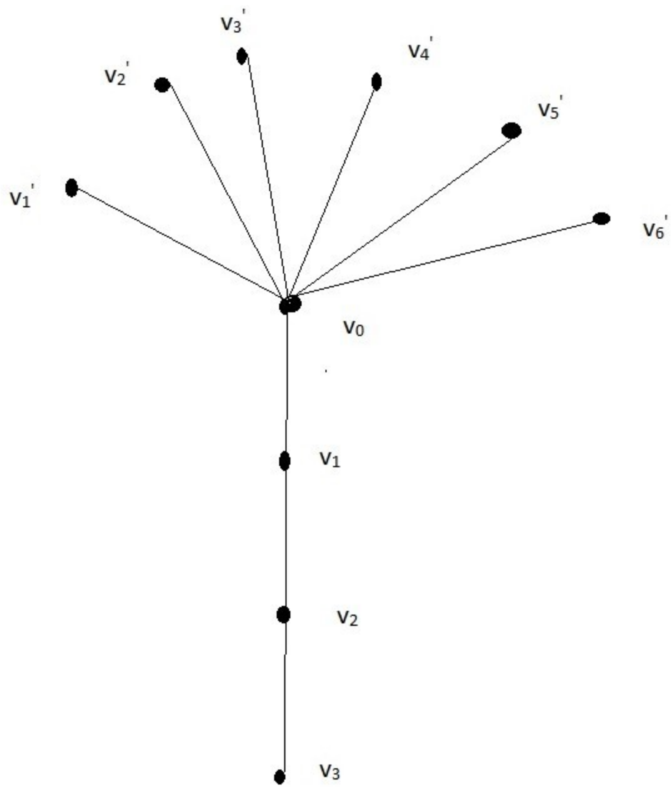


Theorem 3.5. *The Coconut tree $CT(mn)$ is a vertex graceful and Difference Labeling Graph.*

Proof. Let $V = \{v_0, v_1, v_2 \ldots v_n, v_1^{'}, v_2^{'} \ldots v_n^{'}\}$ where v_0 be the centre vertex and $v_1^{'}, v_2^{'} \ldots v_n^{'}\}$ be the pendent vertex which is incident to V_0.
The coconut tree is identifyingthe central vertex of $K_{1,n}$ with a pendent vertex of the path P_m
The coconut tree $CT(m,n)$ is the tree whose vertex set is $V = \{v_0, v_1 \ldots v_n\} \cup \{v_1^{'}, v_2^{'} \ldots v_n^{'}\}$ and edge set is

$E = \{e_i = v_{i+1}v_i, 0 \leq i \leq n\} \cup \{e_i^{'} = v_{i+1}^{'}v_i^{'}, 0 \leq i \leq n\}$.
Now define the function f on vertices V as follows:
$f(v_{i+n}) = i + 1; 0 \leq i \leq n$
$f(v_{i+1}^{'}) = f(v_i) + f(v_n); 0 \leq i \leq n$
Now we can define the edge value as $f^*(uv) = |f(u) - f(v)|$.
Since all the vertices are distinct with two different sets of edges value, coconut tree $CT(m, n)$ is vertex graceful and Difference Labeling Graph. □

Example 3.5. *Vertex Graceful and Difference Labeling Graph of* $CT_{4,6}$ *Graph*



4. Conclusion

In this paper gear graph G_n,swasthick graph S_n, coconut tree graph CT_n, swasthick graph S_n, $\prod$ tree, Y tree are investigated for vertex graceful and Difference Labeling Graph.

5. Scope of Futher Study

On gracefulness of the square and cube difference labeling for above graph may be studied.

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