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Novel concept of ideal nanotopological spaces

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Abstract: In this paper is to introduce and study the concepts of $\mathcal{H}^\star$-I_n-sets, I_n-$\mathcal{O}$-sets, $\mathcal{H}^{\star\star}$-I_n-sets, I_n-$\mathcal{O}_p^\star$-sets, $\mathcal{O}^{\star\star}$-I_n-sets and $\mathcal{O}_p^\star$-I_n-sets in ideal nanotopological spaces.

Key words: $\mathcal{H}^\star$-I_n-set, I_n-$\mathcal{O}$-set, $\mathcal{O}_p^\star$-I_n-set, pre*-I_n-open set, semi*-I_n-open set, $\alpha^\star$-I_n-open set

1. Introduction

The concept of nano topology was introduced by Lellis Thivagar et al (2013). which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure.

K. Bhuvaneswari et al (2014) introduced and studied the concept of nano generalised closed sets in nano topological spaces.

M. Parimala et.al (2017) introduced nano ideal generalized closed sets further developed. Rajasekaran el.al (2018) introduced Simple forms of nano open sets in an ideal nano topological spaces. Nethaji et.al (2019)introduced smei*-I_n-open stes in ideal nanotopological spaces.

In this paper, we introduce and study the notions of $\mathcal{H}^\star$-I_n-sets, I_n-$\mathcal{O}$-sets, $\mathcal{H}^{\star\star}$-I_n-sets, I_n-$\mathcal{O}_p^\star$-sets, $\mathcal{O}^{\star\star}$-I_n-sets and $\mathcal{O}_p^\star$-I_n-sets in an ideal nanotopological spaces.

2. Preliminaries

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $I_n(A)$ and $C_n(A)$, respectively.

we denote a ideal nanotopological space by $(U, \mathcal{N}, I)$. The nano-interior and nano-closure of a subset A of U are denoted by $I_n^\star(A)$ and $C_n^\star(A)$, respectively

Proposition 2.1. *[5] A subset A of a space $(U, \mathcal{N}, I)$ is semi-I_n-open $\iff$ $C_n(A) = C_n(I_n^\star(A))$.*

Proposition 2.2. *[10] The intersection of a pre-I_n-open and n-open is pre-I_n-open.*

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Lemma 2.1. *[10] Let $(U, \mathcal{N}, I)$ be a space and A subset of U. If H is n-open in $(U, \mathcal{N}, I)$, then $H \cap C_n^\star(A) \subset C_n^\star(H \cap A)$.*

Definition 2.1. [7] A subset A of a space $(U, \mathcal{N}, I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $A \subseteq A_n^\star$ (resp. $A = A_n^\star$, $A_n^\star \subseteq A$).

Definition 2.2. A subset A of a space $(U, \mathcal{N})$, is called a

1. n-dense [12] if $C_n(A) = U$,
2. n-submaximal [12] if each n-dense subset of U is n-open.
3. nano nowhere dense (briefly, n-nowhere dense) [3] if $I_n(C_n(A)) = \phi$.
4. nano locally closed (briefly, n-LC) [1] if $A = H \cap K$, where H is n-open and K is n-closed.

3. On novel subsets of $(U, \mathcal{N}, I)$

Definition 3.1. A subset A of a space $(U, \mathcal{N}, I)$ is called a

1. nano pre*-I-closed (written in short as pre*-I_n-closed) if $C_n^\star(I_n(A)) \subset A$.
2. nano $\alpha^\star$-I-closed (written in short as $\alpha^\star$-I_n-closed) if $C_n^\star(I_n(C_n^\star(A))) \subset A$.
3. nano I-t-set (written in short as I_n-t-set) if $I_n^\star(A) = I_n^\star(C_n(A))$.
4. nano pre*-I-regular (written in short as pre*-I_n-regular) if A is pre-I_n-open and pre*-I_n-closed
5. nano $\mathcal{R}^\star$-I-set (written in short as $\mathcal{R}^\star$-I_n-set) if $A = P \cap Q$, where P is $n\star$-open set and Q is I_n-t-set

Example 3.1. *Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{2\}, \{2, 3\}\}$ and $X = \{1, 3\}$. After that $\mathcal{N} = \{\phi, \{1\}, \{2, 3\}, \{1, 2, 3\}, U\}$. Let the ideal be $I = \{\phi, \{3\}\}$.*

1. *$A = \{1, 4\}$ is pre*-I_n-closed, $\alpha^\star$-I_n-closed, I_n-t-set and $\mathcal{R}^\star$-I_n-set.*
2. *$B = \{1, 2, 4\}$ is pre*-I_n-regular.*

Proposition 3.1. *In a space $(U, \mathcal{N}, I)$,*

1. $n\star$-open $\Rightarrow$ pre*-I_n-open.
2. n-open $\Rightarrow$ pre*-I_n-open.

Proof. (i) Let A be a $n\star$-open set in U. After that $A = I_n^\star(A) \subset I_n^\star(C_n(A))$. As a result A is pre*-I_n-open in U.

(ii) Let A be a n-open set. After that A is $n\star$-open. By (i), A is pre*-I_n-open.

Example 3.2. *In Example 3.1,*

1. *$A = \{3\}$ is not $n\star$-open but pre*-I_n-open.*
2. *$B = \{1, 2\}$ is not n-open but pre*-I_n-open.*

Proposition 3.2. *In a space $(U, \mathcal{N}, I)$, each nano pre-open set is pre*-I_n-open.*

Proof. Let A be nano pre-open set in U. After that $A \subset I_n(C_n(A)) \subset I_n^\star(C_n(A))$. As a result A is pre*-I_n-open in U.

Example 3.3. *Let $U = \{1,2,3,4\}$ with $U/R = \{\{2\},\{4\},\{1,3\}\}$ and $X = \{3,4\}$. After that $\mathcal{N} = \{\phi,\{4\},\{1,3\},\{1,3,4\},U\}$. Let the ideal be $I = \{\phi,\{4\}\}$. As a result $\{1,2\}$ is not nano pre-open but pre*-I_n-open .*

Proposition 3.3. *In a space $(U,\mathcal{N},I)$, a subset A of U is semi-I_n-closed in U $\Longleftrightarrow$ A is I_n-t-set in U.*

Proof. A is semi-I_n-closed in U $\Longleftrightarrow$ $U - A$ is semi-I_n-open in U $\Longleftrightarrow$ $C_n^\star(U-A) = C_n^\star(I_n(U-A))$ by Proposition 2.1 $\Longleftrightarrow$ $U - I_n^\star(A) = C_n^\star(U - C_n(A)) = U - I_n^\star(C_n(A))$ $\Longleftrightarrow$ $I_n^\star(A) = I_n^\star(C_n(A))$ $\Longleftrightarrow$ A is I_n-t-set in U.

Remark 3.1. *In a space $(U,\mathcal{N},I)$,*

1. *$n\star$-open set $\Longrightarrow$ $\mathcal{R}^\star$-I_n-set.*
2. *each I_n-t-set $\Longrightarrow$ $\mathcal{R}^\star$-I_n-set.*

Remark 3.2. *The converses of the Remark 3.1 is not true as shown in the coming Example.*

Example 3.4. *In Example 3.1,*

1. *$A = \{4\}$ is not $n\star$-open but $\mathcal{R}^\star$-I_n-set.*
2. *$B = \{1,2\}$ is not I_n-t-set but $\mathcal{R}^\star$-I_n-set.*

Proposition 3.4. *For a subset A of a space $(U,\mathcal{N},I)$, the following are equivalent:*

1. *A is $n\star$-open.*
2. *A is pre*-I_n-open & $\mathcal{R}^\star$-I_n-set.*

Proof. (1) $\Rightarrow$ (2): (2) follows by Proposition 3.1(1) and Remark 3.1(1).
(2) $\Rightarrow$ (1): Given A is $\mathcal{R}^\star$-I_n-set. So $A = P \cap Q$ where P is $n\star$-open and $I_n^\star(Q) = I_n^\star(C_n(Q))$. After that $A \subset P = I_n^\star(P)$. Furthermore A is pre*-I_n-open implies $A \subset I_n^\star(C_n(A)) \subset I_n^\star(C_n(Q)) = I_n^\star(Q)$ by assumption. Thus $A \subset I_n^\star(P) \cap I_n^\star(Q) = I_n^\star(P \cap Q) = I_n^\star(A)$ and hence A is $n\star$-open.

Remark 3.3. *In a space the family of pre*-I_n-open sets & the family of $\mathcal{R}^\star$-I_n-sets are independent.*

Example 3.5. *In Example 3.1,*

1. *$A = \{3\}$ is not $\mathcal{R}^\star$-I_n-set but pre*-I_n-open.*
2. *$B = \{4\}$ is not pre*-I_n-open but $\mathcal{R}^\star$-I_n-set.*

Proposition 3.5. *In a space $(U,\mathcal{N},I)$,*

1. *$n\star$-open $\Longrightarrow$ $\alpha^\star$-I_n-open.*
2. *n-open $\Longrightarrow$ $\alpha^\star$-I_n-open.*

Proof. (1) Let A be a $n\star$-open set in U. After that $A = I_n^\star(A) \subset C_n(I_n^\star(A))$. It implies that $A = I_n^\star(A) \subset I_n^\star(C_n(I_n^\star(A)))$. As a result A is $\alpha^\star$-I_n-open in U.
(2) Let A be n-open set U. After that A is $n\star$-open. By (1), A is $\alpha^\star$-I_n-open.

Example 3.6. *In Example 3.1,*

1. $A = \{3, 4\}$ *is not* $n\star$*-open but* $\alpha^\star$*-*I_n*-open.*
2. $B = \{1, 2, 4\}$ *is not* n*-open but* $\alpha^\star$*-*I_n*-open.*

Proposition 3.6. *In a space* $(U, \mathcal{N}, I)$*, each* $\alpha^\star$*-*I_n*-open is pre***-*I_n*-open.*

Proof. Let A be a $\alpha^\star$-I_n-open set in U. After that $A \subset I_n^\star(C_n(I_n^\star(A))) \subset I_n^\star(C_n(A))$. As a result A is pre*-I_n-open in U.

Example 3.7. *In Example 3.1, then* $\{3\}$ *is not* $\alpha^\star$*-*I_n*-open but pre***-*I_n*-open.*

Proposition 3.7. *In a space* $(U, \mathcal{N}, I)$*, each nano* α*-open is* $\alpha^\star$*-*I_n*-open.*

Proof. Let A be a nano α-open set in U. After that $A \subset I_n(C_n(I_n(A))) \subset I_n^\star(C_n(I_n^\star(A)))$. As a result A is a $\alpha^\star$-I_n-open set in U.

Example 3.8. *In Example 3.1, then* $\{2\}$ *is not nano* α*-open but* $\alpha^\star$*-*I_n*-open.*

Proposition 3.8. *Let* A *be a subset of a space* $(U, \mathcal{N}, I)$*.*

1. *If* $A = C_n^\star(I_n(A))$*, After that* A *is* $\alpha^\star$*-*I_n*-closed in U.*
2. *If* $A = n\text{-}cl(I_n(A))$*, After that* A *is* $\alpha^\star$*-*I_n*-closed in* U*.*

Proof. (1) If $A = C_n^\star(I_n(A))$, after that we obtain that $C_n^\star(I_n(C_n^\star(A))) = C_n^\star(I_n(C_n^\star(I_n(A)))) = C_n^\star(I_n(A)) = A$. As a result A is a $\alpha^\star$-I_n-closed set in U.
(2) If $A = C_n(I_n(A))$, after that we obtain that $C_n^\star(I_n(C_n^\star(A))) = C_n^\star(I_n(C_n^\star(C_n(I_n(A))))) \subset C_n(I_n(C_n(I_n(A)))) = C_n(I_n(A)) = A$. As a result A is an $\alpha^\star$-I_n-closed set in U.

Remark 3.4. *In a space* $(U, \mathcal{N}, I)$*,*

1. *pre***-*I_n*-regular* $\Longrightarrow$ *pre-*I_n*-open.*
2. *pre***-*I_n*-regular* $\Longrightarrow$ *pre***-*I_n*-closed.*

The converses of Remark 3.4 is not true as shown in the coming Examples.

Example 3.9. *In Example 3.1,*

1. $A = \{3\}$ *is not pre***-*I_n*-regular but pre-*I_n*-open.*
2. $B = \{4\}$ *is not pre***-*I_n*-regular but pre***-*I_n*-closed.*

Proposition 3.9. *In a space* $(U, \mathcal{N}, I)$*, each* $\alpha^\star$*-*I_n*-open set is semi***-*I_n*-open.*

Proof. Let A be a $\alpha^\star$-I_n-open set in U. Then $A \subset I_n^\star(C_n(I_n^\star(A))) \subset C_n(I_n^\star(A))$. Thus A is semi*-I_n-open in U.

Example 3.10. *In Example 3.1, then* $\{1,4\}$ *is not* $\alpha^\star$-I_n*-open but semi**-I_n*-open.*

Theorem 3.1. *Let* A *be a subset of a space* $(U,\mathcal{N},I)$*. After that* A *is* $\alpha^\star$-I_n*-open* $\iff$ A *is semi**-I_n*-open and pre**-I_n*-open.*

Proof. Let A be a $\alpha^\star$-I_n-open set in U. Then $A \subset I_n^\star(C_n(I_n^\star(A)))$. It follows that $A \subset C_n(I_n^\star(A))$ and $A \subset I_n^\star(C_n(A))$. As a result, A is semi*-I_n-open and pre*-I_n-open.

Conversely, suppose that A is semi*-I_n-open and pre*-I_n-open in U. After that $A \subset C_n(I_n^\star(A))$ and $A \subset I_n^\star(C_n(A))$. It follows that $A \subset I_n^\star(C_n(A)) \subset I_n^\star(C_n(I_n^\star(A)))$ which implies that A is $\alpha^\star$-I_n-open in U.

Remark 3.5. *In a space the family of semi**-I_n*-open sets and the family of pre**-I_n*-open sets are independent.*

Example 3.11. *In Example 3.1,*

1. $A = \{1,4\}$ *is not pre**-I_n*-open but semi**-I_n*-open.*
2. $B = \{3\}$ *is not semi**-I_n*-open but pre**-I_n*-open.*

4. On novel an ideal nano sets

Definition 4.1. A subset A of a space $(U,\mathcal{N},I)$ is called a

1. nano $\mathcal{O}_p^\star$-I-set (written in short as $\mathcal{O}_p^\star$-I_n-set) if $A = P \cap Q$, where P is n-open and Q is pre*-I_n-regular set.
2. nano $\mathcal{O}^{\star\star}$-I-set (written in short as $\mathcal{O}^{\star\star}$-I_n-set) if $A = P \cap Q$, where P is $n\star$-open and Q is pre*-I_n-regular set.
3. nano I-$\mathcal{O}$-set (written in short as I_n-$\mathcal{O}$-set) if $A = P \cap Q$, where P is n-open and Q is pre*-I_n-closed.
4. nano I-$\mathcal{O}_p^\star$-set (written in short as I_n-$\mathcal{O}_p^\star$-set) if $A = P \cap Q$, where P is $n\star$-open and Q is pre*-I_n-closed.
5. nano α-I-set (written in short as α-I_n-set) if $A = P \cap Q$, where P is n-open and Q is $\alpha^\star$-I_n-closed.
6. nano $\alpha^\star$-I-set (written in short as $\alpha^\star$-I_n-set) if $A = P \cap Q$, where P is $n\star$-open and Q is $\alpha^\star$-I_n-closed.
7. nano $\mathcal{H}^\star$-I-set (written in short as $\mathcal{H}^\star$-I_n-set) if $A = P \cap Q$, where P is n-open and $Q = C_n^\star(I_n(Q))$.
8. nano $\mathcal{H}^{\star\star}$-I-set (written in short as $\mathcal{H}^{\star\star}$-I_n-set) if $A = P \cap Q$, where P is an $n\star$-open and $Q = C_n(I_n(Q))$.

Example 4.1. *In Example 3.3, after that* $\{1,3\}$ *is* $\mathcal{O}_p^\star$-I_n*-set,* $\mathcal{O}^{\star\star}$-I_n*-set,* I_n-$\mathcal{O}$*-set,* I_n-$\mathcal{O}_p^\star$*-set,* α-I_n*-set,* $\alpha^\star$-I_n*-set,* $\mathcal{H}^\star$-I_n*-set and* $\mathcal{H}^{\star\star}$-I_n*-set.*

Remark 4.1. *In a space* $(U,\mathcal{N},I)$*,*

1. *each* n*-open set is* $\mathcal{O}_p^\star$-I_n*-set.*
2. *each pre**-I_n*-regular set is* $\mathcal{O}_p^\star$-I_n*-set.*

The converses of Remark 4.1 is not true as shown in the following Example.

Example 4.2. *In Example 3.3,*

1. $A = \{3,4\}$ *is not* n*-open but* $\mathcal{O}_p^\star$-I_n*-set.*
2. $B = \{1,3\}$ *is not pre**-I_n*-regular but* $\mathcal{O}_p^\star$-I_n*-set.*

Theorem 4.1. *In a space $(U, \mathcal{N}, I)$, each $\mathcal{O}_p^\star$-I_n-set is pre-I_n-open.*

Proof. Let A be $\mathcal{O}_p^\star$-I_n-set in U. It follows that $A = P \cap Q$, where P is n-open and Q is pre*-I_n-regular in U. By Remark 3.4(1), Q is pre-I_n-open. Since Q is pre-I_n-open, by Proposition 2.2, $A = P \cap Q$ is pre-I_n-open set in U.

Example 4.3. *Let $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1, 3\}$. After that $\mathcal{N} = \{\phi, \{2, 3\}, U\}$. Let the ideal be $I = \{\phi, \{2\}\}$. As a result $\{1, 3\}$ is not $\mathcal{O}_p^\star$-I_n-set but pre-I_n-open.*

Remark 4.2. *The following diagram holds for any subset of a space $(U, \mathcal{N}, I)$.*

$$\begin{array}{ccc} \text{pre}^\star\text{-}I_n\text{-regular} & & \\ \downarrow & & \\ \mathcal{O}_p^\star\text{-}I_n\text{-set} & \longrightarrow & \text{pre-}I_n\text{-open} \end{array}$$

Remark 4.3. *In a space $(U, \mathcal{N}, I)$,*

1. *each pre*-I_n-closed is I_n-$\mathcal{O}$-set.*
2. *each $\alpha^\star$-I_n-closed is a α-I_n-set.*
3. *for a subset A of U if $A = C_n^\star(I_n(A))$, then A is $\mathcal{H}^\star$-I_n-set.*

Example 4.4. *In Example 3.3,*

1. *$A = \{1, 3, 4\}$ is not pre*-I_n-closed but I_n-$\mathcal{O}$-set.*
2. *$B = \{1, 3\}$ is not $\alpha^\star$-I_n-closed but α-I_n-set.*

Remark 4.4. *In a space $(U, \mathcal{N}, I)$, each n-open set is $\mathcal{H}^\star$-I_n-set.*

Example 4.5. *In Example 3.3, then $\{1, 2, 3\}$ is not n-open but $\mathcal{H}^\star$-I_n-set.*

Remark 4.5. *The following diagram holds for any subset of a space $(U, \mathcal{N}, I)$.*

$$\begin{array}{ccc} \mathcal{O}_p^\star\text{-}I_n\textbf{-set} & \longrightarrow & I_n\text{-}\mathcal{O}\textbf{-set} \\ & & \uparrow \\ \mathcal{H}^\star\text{-}I_n\textbf{-set} & \longrightarrow & \alpha\text{-}I_n\textbf{-set} \end{array}$$

Remark 4.6. *The reverse implications in Remark 4.5 are not true as shown in the following Example.*

Example 4.6. *In Example 3.3,*

1. *$A = \{2\}$ is not $\mathcal{O}_p^\star$-I_n-set but I_n-$\mathcal{O}$-set.*
2. *$B = \{1, 2, 3\}$ is not α-I_n-set but I_n-$\mathcal{O}$-set.*
3. *$C = \{2, 4\}$ is not $\mathcal{H}^\star$-I_n-set but α-I_n-set.*

Theorem 4.2. *For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent:*

1. *A is I_n-$\mathcal{O}$-set and semi-I_n-open in U.*
2. *$A = P \cap C_n^\star(I_n(A))$ for n-open P.*

Proof. (1) $\Rightarrow$ (2) : Since A is I_n-$\mathcal{O}$-set, $A = P \cap Q$, where P is n-open and Q is pre*-I_n-closed in U. We have $A \subset P$ and $C_n^\star(I_n(A)) \subset C_n^\star(I_n(P)) \subset P$ since P is pre*-I_n-closed in U. Since A is semi-I_n-open in U, we have $A \subset C_n^\star(I_n(A))$. It follows that $A = A \cap C_n^\star(I_n(A)) = P \cap Q \cap C_n^\star(I_n(A)) = P \cap C_n^\star(I_n(A))$.

(2) $\Rightarrow$ (1): Let $A = P \cap C_n^\star(I_n(A))$ for n-open set P. Then $A \subset C_n^\star(I_n(A))$ and thus A is semi-I_n-open in U. Since $C_n^\star(I_n(A))$ is $n\star$-closed set, by Proposition 3.1(1), it is pre*-I_n-closed set in U. As a result, A is I_n-$\mathcal{O}$-set in U.

Theorem 4.3. *For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent:*

1. *A is $\mathcal{H}^\star$-I_n-set.*
2. *A is semi-I_n-open and α-I_n-set.*
3. *A is semi-I_n-open and I_n-$\mathcal{O}$-set.*

Proof. (1) $\Rightarrow$ (2): Suppose that A is $\mathcal{H}^\star$-I_n-set in U. It follows that $A = P \cap Q$, where P is n-open set and $Q = C_n^\star(I_n(Q))$. This implies that $A = P \cap Q = P \cap C_n^\star(I_n(Q)) \subset C_n^\star(P \cap I_n(Q))$ (by Lemma 2.1) $= C_n^\star(int(P) \cap I_n(Q)) \subset C_n^\star(I_n(P) \cap I_n(Q)) = C_n^\star(I_n(P \cap Q)) = C_n^\star(I_n(A))$. As a result $A \subset C_n^\star(I_n(A))$ and hence A is semi-I_n-open set. Moreover, by Remark 4.5, A is α-I_n-set in U.

(2) $\Rightarrow$ (3): It follows from the fact that each α-I_n-set is I_n-$\mathcal{O}$-set in U by Remark 4.5.

(3) $\Rightarrow$ (1): Suppose A is semi-I_n-open and I_n-$\mathcal{O}$-set in U. By Theorem 4.2, $A = P \cap C_n^\star(I_n(A))$ for n-open set P. We have $C_n^\star(I_n(C_n^\star(I_n(A)))) = C_n^\star(I_n(A))$. It follows that A is $\mathcal{H}^\star$-I_n-set in U.

Remark 4.7. *In a space $(U, \mathcal{N}, I)$,*

1. *each $n\star$-open set is $\mathcal{O}^{\star\star}$-I_n-set.*
2. *each pre*-I_n-regular set is $\mathcal{O}^{\star\star}$-I_n-set.*

The converses of Remark 4.7 are not true as shown in the coming Example.

Example 4.7. *In Example 3.3,*

1. *$A = \{1, 4\}$ is not $n\star$-open but $\mathcal{O}^{\star\star}$-I_n-set.*
2. *$B = \{1, 3, 4\}$ is not pre*-I_n-regular but $\mathcal{O}^{\star\star}$-I_n-set.*

Remark 4.8. *In a space $(U, \mathcal{N}, I)$,*

1. *each pre*-I_n-closed is I_n-$\mathcal{O}_p^\star$-set.*
2. *each $\alpha^\star$-I_n-closed is $\alpha^\star$-I_n-set.*
3. *For a subset A of U if $A = C_n(I_n(A))$, as a result A is $\mathcal{H}^{\star\star}$-I_n-set.*

Example 4.8. *In Example 3.3,*

1. *$A = \{2\}$ is not pre*-I_n-closed but I_n-$\mathcal{O}_p^\star$-set.*
2. *$B = \{1, 3, 4\}$ is not $\alpha^\star$-I_n-closed but $\alpha^\star$-I_n-set.*

Remark 4.9. *The following diagram holds for any subset of a space $(U, \mathcal{N}, I)$.*

$$\begin{array}{ccc} \mathcal{O}^{\star\star}\text{-}I_n\textbf{-set} & \longrightarrow & I_n\text{-}\mathcal{O}_p^{\star}\textbf{-set} \\ & & \uparrow \\ \mathcal{H}^{\star\star}\text{-}I_n\textbf{-set} & \longrightarrow & \alpha^{\star}\text{-}I_n\textbf{-set} \end{array}$$

Remark 4.10. *The reverse implications in Remark 4.9 are not true as shown in the coming Example.*

Example 4.9. *In Example 3.3,*

1. $A = \{2\}$ *is not* $\mathcal{O}^{\star\star}$-I_n*-set but* I_n-$\mathcal{O}_p^{\star}$*-set.*
2. $B = \{1, 4\}$ *is not* $\alpha^{\star}$-I_n*-set but* I_n-$\mathcal{O}_p^{\star}$*-set.*

Example 4.10. *In Example 4.3, then* $\{4\}$ *is not* $\mathcal{H}^{\star\star}$-I_n*-set but* $\alpha^{\star}$-I_n*-set.*

Remark 4.11. *In a space* $(U, \mathcal{N}, I)$*, each* $n\star$*-open set is* $\mathcal{H}^{\star\star}$-I_n*-set.*

Example 4.11. *In Example 3.3, then* $\{2, 4\}$ *is not* $n\star$*-open but* $\mathcal{H}^{\star\star}$-I_n*-set.*

Proposition 4.1. *For a subset* A *of a space* $(U, \mathcal{N}, I)$*, the following are equivalent:*

1. A *is* $n\star$*-open.*
2. A *is* $\alpha^{\star}$-I_n*-open and* $\mathcal{H}^{\star\star}$-I_n*-set.*
3. A *is pre**-I_n*-open and* $\mathcal{H}^{\star\star}$-I_n*-set.*

Proof. (1) $\Rightarrow$ (2): (2) follows by Proposition 3.5(1) and Remark 4.11.

(2) $\Rightarrow$ (3): (3) follows by Proposition 3.6.

(3) $\Rightarrow$ (1): Suppose A is pre*-I_n-open and $\mathcal{H}^{\star\star}$-I_n-set. Since A is $\mathcal{H}^{\star\star}$-I_n-set, we have $A = P \cap Q$, where P is $n\star$-open set and $Q = C_n(I_n(Q))$. It follows that $I_n^{\star}(C_n(Q)) \subset C_n(Q) \subset C_n(Q) = C_n(C_n(I_n(Q))) = C_n(I_n(Q)) = Q$. This implies that Q is semi-I_n-closed. By Proposition 3.3, Q is I_n-t-set in U. By Definition 3.1(5), A is $\mathcal{R}^{\star}$-I_n-set. Since A is pre*-I_n-open and $\mathcal{R}^{\star}$-I_n-set, A is $n\star$-open by Proposition 3.4.

Remark 4.12. *The coming Example shows that*

1. *In a space the family of* $\alpha^{\star}$-I_n*-open and the family of* $\mathcal{H}^{\star\star}$-I_n*-sets are independent.*
2. *In a space the family of pre**-I_n*-open and the family of* $\mathcal{H}^{\star\star}$-I_n*-sets are independent.*

Example 4.12. *In Example 3.3,*

1. $A = \{2, 3\}$ *is not* $\alpha^{\star}$-I_n*-open but* $\mathcal{H}^{\star\star}$-I_n*-set.*
2. $B = \{2, 4\}$ *is not pre**-I_n*-open but* $\mathcal{H}^{\star\star}$-I_n*-set.*
3. $C = \{1\}$ *is not* $\mathcal{H}^{\star\star}$-I_n*-set but pre**-I_n*-open.*

Example 4.13. *In Example 4.3, then* $\{3\}$ *is not* $\mathcal{H}^{\star\star}$-I_n*-set but* $\alpha^{\star}$-I_n*-open.*

Theorem 4.4. *For a subset* A *of a space* $(U, \mathcal{N}, I)$*, the following are equivalent:*

1. A *is* I_n-$\mathcal{O}_p^{\star}$*-set and semi-*I_n*-open in* U*.*
2. $A = P \cap C_n^{\star}(I_n(A))$ *for* $n\star$*-open in* P*.*

Proof. (1) $\Rightarrow$ (2) : Since A is I_n-$\mathcal{O}_p^\star$-set, $A = P \cap Q$, where P is $n\star$-open and Q is pre*-I_n-closed in U. We have $A \subset P$ and $C_n^\star(I_n(A)) \subset C_n^\star(I_n(P)) \subset P$ since P is pre*-I_n-closed in U. Since A is semi-I_n-open in U, we have $A \subset C_n^\star(I_n(A))$. It follows that $A = A \cap C_n^\star(I_n(A)) = P \cap Q \cap C_n^\star(I_n(A)) = P \cap C_n^\star(I_n(A))$.

(2) $\Rightarrow$ (1): Let $A = P \cap C_n^\star(I_n(A))$ for $n\star$-open in P. Then $A \subset C_n^\star(I_n(A))$ and thus A is semi-I_n-open in U. Since $C_n^\star(I_n(A))$ is $n\star$-closed, by Proposition 3.1(1), it is pre*-I_n-closed in U. Hence, A is I_n-$\mathcal{O}_p^\star$-set in U.

5. Various style of an ideal nano locally closed sets

Definition 5.1. A subset A of a space $(U, \mathcal{N}, I)$ is called

1. nano locally I-closed (written in short as I_n-LC) if $A = P \cap Q$, where P is n-open and Q is n-closed.
2. nano locally $I^\star$-closed (written in short as $I_n^\star$-LC) if $A = P \cap Q$, where P is $n\star$-open and Q is n-closed.
3. nano $\star$-I-locally closed set (written in short as $\star$-I_n-LC) if $A = P \cap Q$ where P is $n\star$-open and Q is $n\star$-closed.
4. weakly nano I-locally closed set (written in short as $\mathcal{W}$-I_n-LC) if $A = P \cap Q$ where P is n-open and Q is $n\star$-closed.
5. nano $I^\star$-submaximal (written in short as $I_n^\star$-submaximal) if every n-dense subset of U is $n\star$-open.

Example 5.1. *In Example 3.3, then $\{1, 3\}$ is I_n-LC, $I_n^\star$-LC, $\star$-I_n-LC, $\mathcal{W}$-I_n-LC and $I_n^\star$-submaximal.*

Remark 5.1. *In a space $(U, \mathcal{N}, I)$,*

1. *each $n\star$-open is $I_n^\star$-LC.*
2. *each n-closed is $I_n^\star$-LC.*

The converses of Remark 5.1 are not true as shown in the coming Example.

Example 5.2. *In Example 3.3,*

1. *$A = \{2\}$ is not $n\star$-open but $I_n^\star$-LC.*
2. *$B = \{1, 3\}$ is not n-closed but $I_n^\star$-LC.*

Theorem 5.1. *For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent,*

1. *A is $\mathcal{W}$-I_n-LC,*
2. *$A = G \cap C_n^\star(A)$ for few n-open in G,*
3. *$C_n^\star(A) - A = A_n^\star - A$ is n-closed,*
4. *$(U - A_n^\star) \cup A = A \cup (U - C_n^\star(A))$ is n-open,*
5. *$A \subset I_n(A \cup (U - A_n^\star))$.*

Proof. (1) $\Rightarrow$ (2) If A is $\mathcal{W}$-I_n-LC, then there exist n-open G and $n\star$-closed, K such that $A = G \cap K$. But $A \subset G \cap C_n^\star(A)$. Since K is $n\star$-closed, $C_n^\star(A) \subset C_n^\star(K) = K$ and so $G \cap C_n^\star(A) \subset G \cap K = A$. There fore $A = G \cap C_n^\star(A)$.

(2) ⇒ (3) Now $A_n^\star - A = A_n^\star \cap (U - A) = A_n^\star \cap (U - (G \cap n\text{-}cl^\star(A))) = A_n^\star \cap (U - G)$. Therefore, $A_n^\star - A$ is closed.

(3) ⇒ (4) Since $U - (A_n^\star - A) = (U - A_n^\star) \cup A$, $(U - A_n^\star) \cup A$ is n-open. But, $(U - A_n^\star) \cup A = A \cup (U - C_n^\star(A))$.

(4) ⇒ (5) is clear.

(5) ⇒ (1) $U - A_n^\star = I_n(U - A_n^\star) \subset I_n(A \cup (U - A_n^\star))$ which implies that $A \cup (U - A_n^\star) \subset I_n(A \cup (U - A_n^\star))$ and so $A \cup (U - A_n^\star)$ is n-open. Since $A = (A \cup (U - A_n^\star)) \cap C_n^\star(A)$, A is $\mathcal{W}$-I_n-LC.

Proposition 5.1. *For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent:*

1. *A is $n\star$-open.*
2. *A is pre*-I_n-open and $I_n^\star$-LC.*

Proof. (1) ⇒ (2): (2) follows by Proposition 3.1(1) and Remark 5.1(1).
(2) ⇒ (1): Given A is locally $I_n^\star$-closed. So $A = P \cap Q$ where P is $n\star$-open and $Q = C_n(Q)$. Then $A \subset P = I_n^\star(P)$. Also A is pre*-I_n-open implies $A \subset I_n^\star(C_n(A)) \subset I_n^\star(C_n(Q)) = I_n^\star(Q)$ by assumption. Thus $A \subset I_n^\star(P) \cap I_n^\star(Q) = I_n^\star(P \cap Q) = I_n^\star(A)$ and hence A is $n\star$-open.

Remark 5.2. *In a space the family of pre*-I_n-open and the family of locally $I_n^\star$-closed($I_n^\star$-LC)sets are independent.*

Example 5.3. *In Example 3.3,*

1. *$A = \{1\}$ is not $I_n^\star$-LC but pre*-I_n-open.*
2. *$B = \{2\}$ is not pre*-I_n-open but $I_n^\star$-LC.*

Proposition 5.2. *In a space $(U, \mathcal{N}, I)$, each n-LC is $I_n^\star$-LC.*

Proof. It follows from the facts that n-closed and each n-open is $n\star$-open.
The converse of Proposition 5.2 is not true as shown in the coming Example.

Example 5.4. *In Example 4.3, then $\{3\}$ is not n-LC but $I_n^\star$-LC.*

Proposition 5.3. *In a space $(U, \mathcal{N}, I)$, each n-submaximal is $I_n^\star$-submaximal.*

Proof. Let A be n-dense in U. After that $U = C_n(A) \subset C_n(A)$ and $U = C_n(A)$. Thus A is n-dense in U. Since U is n-submaximal, A is n-open and hence $n\star$-open in U. Hence, U is $I_n^\star$-submaximal.

Example 5.5. *In Example 3.3, then $\{4\}$ is not n-submaximal but $I_n^\star$-submaximal.*

Proposition 5.4. *In a space $(U, \mathcal{N}, I)$, each $n\star$-open set is semi*-I_n-open.*

Proof. Let A be $n\star$-open set in U. After that $A = I_n^\star(A) \subset C_n(I_n^\star(A))$. As a rsult A is semi*-I_n-open in U.

Example 5.6. *In Example 3.3, then $\{2, 4\}$ is not $n\star$-open but semi*-I_n-open.*

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