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# Nano generalized closed sets depending on ideal

**I. Rajasekaran[1*], O. Nethaji[2], R. Asokan[3] and R. Premkumar[4]**

[1]Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam - 627 113, Tirunelveli, Tamil Nadu, India. Orchid iD: 0000-0001-8528-4396

[2]Research Scholar, School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India. Orchid iD: 0000-0002-2004-3521

[3]Department of Mathematics, School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India. Orchid iD: 0000-0002-0992-1426

[4]Department of Mathematics, Senthamarai College of Arts and Science,Vadapalanji-21, Madurai, Tamil Nadu, India. Orchid iD: 0000-0002-8088-5114

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**Abstract:** In this paper, we made an attempt to notions of $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets. Also we characterize the relations between them and the related properties.

**Key words:** $n$-open, $n$-closed, Ideal and $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed

## 1. Introduction

During this paper, R. Vaidyanathaswamy [5, 6] and K. Kuratowski [2] Independly introduced the concept an ideals. Z. Pawlak [4] introduced lower approximation, upper approximation and boundary are investigated and M. Lellis Thivagar et al. [3] introduced the notions of weaker forms of nanotopological spaces and K. Bhuvaneshwari et al [1] introduced the nano generalized closed sets.

In this paper, we made an attempt to notions of $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets. Also we characterize the relations between them and the related properties.

In this article use the notions, "$C_n$" denotes the nano closure operator and "$I_n$" denotes the nano interior operator.

## 2. Nano $g$-closed sets depending on ideal

**Definition 2.1.** Let $(U,\mathcal{N})$ be a space and $I$ be an ideal on $U$. A subset $D$ of $U$ is called

1. nano $g$-closed sets depending on ideal (written in short as $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed) $\Longleftrightarrow$ $C_n(D) \cap \mathcal{K} \in I$ (where $\mathcal{K}$ denotes the complement), when $D \subseteq K$ and $K$ is $n$-open.
2. nano $g$-open sets depending on ideal (written in short as $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open) $\Longleftrightarrow$ $U - D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.

**Theorem 2.1.** *Let $(U,\mathcal{N})$ be a space. Then the conditions are equivalent.*


*Correspondence: sekarmelakkal@gmail.com

1. *$D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $(U, \mathcal{N})$,*

2. *$S \subseteq C_n(D) - D$ and $S$ is $n$-closed in $U$ implies $S \in I$.*

*Proof.* (1) ⇒ (2) Suppose by (1). Let $S \subseteq C_n(D) - D$. Suppose $S$ is $n$-closed. Then $D \subseteq U - S$. By our assumption, $C_n(D) - (U - S) \in I$. But $S \subseteq C_n(D) - (U - S)$ and therefore $S \in I$.

(2) ⇒ (1) Assume that by (2). Suppose $D \subseteq J$ & $J$ is $n$-open. Then $C_n(D) - J = C_n(D) \cap (U - J)$ is a $n$-closed set in $U \subseteq C_n(D) - D$.

By hypothesis, $C_n(D) - J \in I \Longrightarrow D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.

**Remark 2.1.** *Each $ng$-closed set is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed but not converse as shown in the next Example.*

**Example 2.1.** *Let $U = \{m_1, m_2, m_3\}$ with $U/R = \{\{m_2\}, \{m_1, m_3\}\}$ and $X = \{m_2\}$. Then $\mathcal{N} = \{\phi, U, \{m_2\}\}$. Let the ideal be $I = \{\phi, \{m_1\}, \{m_3\}, \{m_1, m_3\}\}$, then $A = \{m_2\}$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed need not to be $ng$-closed.*

**Theorem 2.2.** *If $D$ and $K$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets of $(U, \mathcal{N})$, then their union $D \cup K$ is also $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.*

*Proof.* Suppose $D$ and $K$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets in $(U, \mathcal{N})$. If $D \cup K \subseteq J$ and $J$ is $n$-open, then $D \subseteq J$ and $K \subseteq J$. By assumption, $C_n(D) - J$ in $I$ and $C_n(K) - J \in I$ and therefore $C_n(D \cup K) - J = (C_n(D) - J) \cup (C_n(K) - J) \in I$. That is $D \cup K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.

**Remark 2.2.** *If $D$ and $J$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets, $D \cap J$ is not $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed as shown by the next Example.*

**Example 2.2.** *Let $U = \{a_1, a_2, a_3\}$ with $U/R = \{\{a_2\}, \{a_1, a_3\}\}$ and $X = \{a_2\}$. Then $\mathcal{N} = \{\phi, U, \{a_2\}\}$. Let the ideal be $I = \{\phi\}$. Then $D = \{a_1, a_2\}$ and $K = \{a_2, a_3\}$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed set. But $D \cap K = \{a_2\}$ is not $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.*

**Theorem 2.3.** *If $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed and $D \subseteq K \subseteq C_n(D)$ in $(U, \mathcal{N})$, then $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.*

*Proof.* Suppose $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed and $D \subseteq K \subseteq C_n(D)$ in $(U, \mathcal{N})$. Suppose $K \subseteq J$ and $J$ is $n$-open. Then $D \subseteq J$. Since $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed, we've $C_n(D) - J \in I$.

At the present $K \subseteq C_n(D)$. This implies that $C_n(K) - J \subseteq C_n(D) - F \in I$. therefore $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $(U, \mathcal{N})$.

**Theorem 2.4.** *Let $D \subseteq V \subseteq U$ and suppose that $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $(U, \mathcal{N})$. Then $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed relative to the subspace $V$ of $U$, depending on the ideal $I_V = \{S \subseteq V : S \in I\}$.*

*Proof.* Suppose $D \subseteq J \cap V$ and $J$ is $n$-open in $(U, \mathcal{N})$, then $D \subseteq J$. Since $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $(U, \mathcal{N})$, we've $C_n(D) - J \in I$. Now $(C_n(D) \cap V) - (F \cap V) = (C_n(D) - F) \cap V \in I$, whenever $D \subseteq J \cap V$ and $J$ is $n$-open. therefore $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed relative to the subspace $V$.

**Theorem 2.5.** *Let $D$ be $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed set and $S$ be a $n$-closed, then $D \cap S$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed set in $(U, \mathcal{N})$.*

*Proof.* Let $D \cap S \subseteq J$ and $J$ is $n$-open. Then $D \subseteq J \cup (U - S)$. Since $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed, we have $C_n(D) - (F \cup (U - S)) \in I$. Now, $C_n(D \cap S) \subseteq C_n(D) \cap S = (C_n(D) \cap S) - (U - S)$. Therefore, $C_n(D \cap S) - J \subseteq (C_n(D) \cap S) - (F \cap (U - S)) \subseteq C_n(D) - (F \cup (U - S)) \in I$. therefore $D \cap S$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $(U, \mathcal{N})$.

**Theorem 2.6.** *Let $(U,\mathcal{N})$ be a space. Then the next conditions are equivalent.*

1. *$D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open in $(U,\mathcal{N})$,*
2. *$S-F \subseteq I_n(D)$, in favor of several $J \in I$, whenever $S \subseteq D$ and $S$ is $n$-closed.*

*Proof.* (1) $\Rightarrow$ (2) Suppose by (1). Suppose $S \subseteq D$ and $S$ is $n$-closed. We have $U-H \subseteq U-S$. By assumption, $C_n(U-D) \subseteq (U-S) \cup J$, in favor of several $J \in I$. This implies $U-((U-S)\cup F) \subseteq U-(C_n(U-D))$ and therefore $S-J \subseteq I_n(D)$.

(2) $\Rightarrow$ (1) Assume that $S \subseteq D$ and $S$ is $n$-closed. Then $S-J \subseteq I_n(D)$, in favor of several $J \in I$. Consider a $n$-open set $P$ such that $U-H \subseteq P$. Then $U-P \subseteq D$. By assumption, $(U-P)-J \subseteq I_n(D) = U-C_n(U-D)$. This gives that $U-(P\cup J) \subseteq U-C_n(U-D)$. Then, $C_n(U-D) \subseteq P \cup J$, in favor of several $J \in I$. This shows that $C_n(U-D)-P \in I$. therefore $U-D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.

Recall that the sets $D$ and $K$ are said to be separated if $C_n(D)\cap K = \phi$ and $D \cap C_n(K) = \phi$.

**Theorem 2.7.** *If $D$ and $K$ are separated $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open sets in $(U,\mathcal{N})$, then $D\cup K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.*

*Proof.* Suppose $D$ and $K$ are separated $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open sets in $(U,\mathcal{N})$ and $S$ be a $n$-closed subset of $D\cup K$. Then $S\cap C_n(D) \subseteq D$ and $S\cap C_n(K) \subseteq K$. By assumption, $(S\cap C_n(D))-J_1 \subseteq I_n(D)$ and $(S\cap C_n(K))-J_2 \subseteq I_n(K)$, in favor of several $J_1$, $J_2 \in I$. It means that $((S\cap C_n(D))-I_n(D)) \in I$ and $((S\cap C_n(K))-I_n(K)) \in I$. Then $((S\cap C_n(D))-I_n(D)) \cup ((S\cap C_n(K))-I_n(K)) \in I$. therefore $(S\cap(C_n(D)\cup C_n(K))-(I_n(D)\cup I_n(K))) \in I$. But $S = S\cap(D\cup K) \subseteq S\cap C_n(D\cup K)$, and we have $S-I_n(D\cup K) \subseteq (S\cap C_n(D\cup K))-I_n(D\cup K) \subseteq (S\cap C_n(D\cup K))-(I_n(D)\cup I_n(K)) \in I$. therefore, $S-J \subseteq I_n(D\cup K)$, in favor of several $J \in I$. This proves that $D\cup K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.

**Corollary 2.1.** *Let $D$ and $K$ be $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed sets and suppose $U-D$ & $U-K$ are separated in $(U,\mathcal{N})$. Then $D\cap K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.*

**Corollary 2.2.** *If $D$ and $K$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open sets in $(U,\mathcal{N})$, then $D\cap K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.*

*Proof.* If $D$ and $K$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open, then $U-D$ and $U-K$ are $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed. By Theorem 2.2, $U-(D\cap K)$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed, which implies $D\cap K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.

**Theorem 2.8.** *If $D \subseteq K \subseteq U$, $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $D$ and $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $U$, then $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $U$.*

*Proof.* Suppose $D \subseteq K \subseteq U$, $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $K$ and $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $U$. Suppose $S \subseteq D$ and $S$ is $n$-closed. Since $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $K$, by theorem 2.6, $S-J_1 \subseteq I_{n_K}(D)$, in favor of several $J_1 \in I$. This implies there exists a $n$-open set $U_1$ such that $F-J_1 \subseteq P_1\cap K \subseteq D$, in favor of several $J_1 \in I$. Since $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open, $S \subseteq K$ and $S$ is $n$-closed; we have $S-J_2 \subseteq I_n(K)$, in favor of several $J_2 \in I$. This implies there exists an open set $P_2$ such that $S-J_2 \subseteq P_2 \subseteq K$, in favor of several $J_2 \in I$. Now $S-(J_1\cup J_2) \subseteq (S-J_1)\cap(S-J_2) \subseteq P_1\cap P_2 \subseteq P_1\cap K \subseteq D$. This implies that $S-(J_1\cup J_2) \subseteq I_n(D)$, in favor of several $J_1\cup J_2 \in I$ and therefore $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open relative to $U$.

**Theorem 2.9.** *If $I_n(D) \subseteq K \subseteq D$ and if $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open in $(U,\mathcal{N})$, then $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open in $U$.*

*Proof.* Suppose $I_n(D) \subseteq K \subseteq D$ and $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open. Then $U - H \subseteq U - K \subseteq C_n(U - D)$ and $U - D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed. By Theorem 2.3, $U - K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed and therefore $K$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.

**Theorem 2.10.** *Let $(U, \mathcal{N})$ be a space, A set $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed in $U \iff C_n(D) - D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open in $U$.*

*Proof.* Necessity: Suppose $S \subseteq C_n(D) - D$ and $S$ be $n$-closed. Then $S \in I$. This implies that $S - J = \phi$, in favor of several $J \in I$. Clearly, $S - J \subseteq I_n(C_n(D) - D)$. By Theorem 2.6, $C_n(D) - D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-open.

Sufficiency: Suppose $D \subseteq P$ and $P$ is $n$-open. Then $C_n(D) \cap (U - P) \subseteq C_n(D) \cap (U - D) = C_n(D) - D$. By hypothesis and $(C_n(D) \cap (U - P)) - J \subseteq I_n(C_n(D) - D) = \phi$, in favor of several $J \in I$. This implies that $C_n(D) \cap (U - P) \subseteq F \in I$ and therefore $C_n(D) - P \in I$. Thus, $D$ is $\mathfrak{n}\mathbb{I}_{\mathfrak{g}}$-closed.

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